

## THE SPECTRUM PROBLEM FOR THE CONNECTED CUBIC GRAPHS OF ORDER 10

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### Abstract

We show that if  $G$  is a connected cubic graph of order 10, then there exists a  $G$ -decomposition of  $K_v$  if and only if  $v \equiv 1$  or  $10 \pmod{15}$  except when  $v = 10$  and  $G$  is one of 5 specific graphs.

**Keywords:** spectrum problem, graph decomposition, cubic graphs.

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