Discussiones Mathematicae
Graph Theory 41 (2021) 963–980
https://doi.org/10.7151/dmgt.2220

THE SPECTRUM PROBLEM FOR THE CONNECTED CUBIC GRAPHS OF ORDER 10

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Abstract

We show that if G is a connected cubic graph of order 10, then there exists a G-decomposition of K_v if and only if $v \equiv 1$ or 10 (mod 15) except when $v = 10$ and G is one of 5 specific graphs.

Keywords: spectrum problem, graph decomposition, cubic graphs.

2010 Mathematics Subject Classification: 05C51, 05C70.

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Received 4 December 2018
Revised 27 March 2019
Accepted 27 March 2019