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SUFFICIENT CONDITIONS FOR A DIGRAPH TO ADMIT A $(1, \leq \ell)$-IDENTIFYING CODE

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Abstract

A $(1, \leq \ell)$-identifying code in a digraph D is a subset C of vertices of D such that all distinct subsets of vertices of cardinality at most ℓ have distinct closed in-neighbourhoods within C. In this paper, we give some sufficient conditions for a digraph of minimum in-degree $\delta^- \geq 1$ to admit a $(1, \leq \ell)$-identifying code for $\ell \in \{\delta^-, \delta^- + 1\}$. As a corollary, we obtain the result by Laihonen that states that a graph of minimum degree $\delta \geq 2$ and girth at least 7 admits a $(1, \leq \delta)$-identifying code. Moreover, we prove that every 1-in-regular digraph has a $(1, \leq 2)$-identifying code if and only if the girth of the digraph is at least 5. We also characterize all the 2-in-regular digraphs admitting a $(1, \leq \ell)$-identifying code for $\ell \in \{2, 3\}$.

Keywords: graph, digraph, identifying code.

2010 Mathematics Subject Classification: 05C69, 05C20.

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