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OUTPATHS OF ARCS IN REGULAR 3-PARTITE TOURNAMENTS

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Abstract

Guo [Outpaths in semicomplete multipartite digraphs, Discrete Appl. Math. 95 (1999) 273–277] proposed the concept of the outpath in digraphs. An outpath of a vertex x (an arc xy, respectively) in a digraph is a directed path starting at x (an arc xy, respectively) such that x does not dominate the end vertex of this directed path. A k-outpath is an outpath of length k. The outpath is a generalization of the directed cycle. A c-partite tournament is an orientation of a complete c-partite graph.

In this paper, we investigate outpaths of arcs in regular 3-partite tournaments. We prove that every arc of an r-regular 3-partite tournament has 2- (when $r \ge 1$), 3- (when $r \ge 2$), and 5-, 6-outpaths (when $r \ge 3$). We also give the structure of an r-regular 3-partite tournament D with $r \ge 2$ that contains arcs which have no 4-outpaths. Based on these results, we conjecture that for all $k \in \{1, 2, \ldots, r-1\}$, every arc of r-regular 3-partite tournaments with $r \ge 2$ has (3k-1)- and 3k-outpaths, and it has a (3k+1)-outpath except an r-regular 3-partite tournament.

Keywords: multipartite tournament, regular 3-partite tournament, outpaths.

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References

- J. Bang-Jensen and G. Gutin, Digraphs: Theory, Algorithms and Applications, 2nd Edition (Springer, London, 2009). https://doi.org/10.1007/978-1-84800-998-1
- [2] L. Cui and Q. Guo, Outpaths of arcs in almost regular multipartite tournaments, Acta Math. Appl. Sin. (Chinese Ser.) 39 (2016) 310–317.

- [3] Q. Guo and L. Cui, Outpaths of all length of an arc in regular multipartite tournaments, Appl. Math. J. Chinese Univ. (Chinese Ser.) 29 (2014) 288–294.
- Y. Guo, Outpaths in semicomplete multipartite digraphs, Discrete Appl. Math. 95 (1999) 273-277. https://doi.org/10.1016/S0166-218X(99)00080-3
- [5] L. Volkmann, Multipartite tournaments: a survey, Discrete Math. 307 (2007) 3097– 3129. https://doi.org/10.1016/j.disc.2007.03.053
- [6] G. Xu, S. Li, Q. Guo and H. Li, Notes on cycles through a vertex or an arc in regular 3-partite tournaments, Appl. Math. Lett. 25 (2012) 662–664. https://doi.org/10.1016/j.aml.2011.09.075
- [7] G. Zhou and K. Zhang, Outpaths of arcs in multipartite tournaments, Acta Math. Appl. Sin. (Engl. Ser.) 17 (2001) 361–365.

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