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A NOTE ON POLYNOMIAL ALGORITHM FOR COST COLORING OF BIPARTITE GRAPHS WITH $\Delta \leq 4$

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Abstract

In the note we consider vertex coloring of a graph in which each color has an associated cost which is incurred each time the color is assigned to a vertex. The cost of coloring is the sum of costs incurred at each vertex. We show that the minimum cost coloring problem for n-vertex bipartite graph of degree $\Delta \leq 4$ can be solved in $O(n^2)$ time. This extends Jansen's result [K. Jansen, *The optimum cost chromatic partition problem*, in: Proc. CIAC'97, Lecture Notes in Comput. Sci. 1203 (1997) 25–36] for paths and cycles to subgraphs of biquartic graphs.

Keywords: bipartite graph, chromatic sum, cost coloring, NP-completeness, polynomial algorithm.

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