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ON SEMISYMMETRIC CUBIC GRAPHS OF ORDER $20p^2$, p PRIME

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Abstract

A simple graph is called semisymmetric if it is regular and edge-transitive but not vertex-transitive. Let p be an arbitrary prime. Folkman proved [*Regular line-symmetric graphs*, J. Combin. Theory 3 (1967) 215–232] that there is no semisymmetric graph of order $2p$ or $2p^2$. In this paper an extension of his result in the case of cubic graphs of order $20p^2$ is given. We prove that there is no connected cubic semisymmetric graph of order $20p^2$ or, equivalently, that every connected cubic edge-transitive graph of order $20p^2$ is necessarily symmetric.

Keywords: edge-transitive graph, vertex-transitive graph, semisymmetric graph, order of a graph, classification of cubic semisymmetric graphs.

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References

[1] M. Alaeiyan and M. Ghasemi, *Cubic edge-transitive graphs of order* $8p^2$, Bull. Aust. Math. Soc. **77** (2008) 315–323. https://doi.org/10.1017/S0004972708000361

[2] M. Alaeiyan and B.N. Onagh, *Cubic edge-transitive graphs of order* $4p^2$, Acta Math. Univ. Comenian. **LXXVIII** (2009) 183–186.

[3] M. Alaeiyan and B.N. Onagh, *On semisymmetric cubic graphs of order* $10p^3$, Hacet. J. Math. Stat. **40** (2011) 531–535.

[4] Y. Bugeand, Z. Cao and M. Mignotte, *On simple K_4-groups*, J. Algebra **241** (2001) 658–668.
https://doi.org/10.1006/jabr.2000.8742

[5] G. Butler and J. McKay, *The transitive groups of degree up to eleven*, Comm. Algebra **11** (1983) 863–911.
https://doi.org/10.1080/00927878308822884

[6] M. Conder, A. Malnič, D. Marušič and P. Potočnik, *A census of semisymmetric cubic graphs on up to* 768 *vertices*, J. Algebraic Combin. **23** (2006) 255–294.
https://doi.org/10.1007/s10801-006-7397-3

[7] M. Conder and R. Nedela, *A refined classification of symmetric cubic graphs*, J. Algebra **322** (2009) 722–740.
https://doi.org/10.1016/j.jalgebra.2009.03.011

[8] Y. Feng, M. Ghasemi and W. Changqun, *Cubic semisymmetric graphs of order* $6p^3$, Discrete Math. **310** (2010) 2345–2355.
https://doi.org/10.1016/j.disc.2010.05.018

[9] J. Folkman, *Regular line-symmetric graphs*, J. Combin. Theory **3** (1967) 215–232.
https://doi.org/10.1016/S0021-9800(67)80069-3

[10] D.M. Goldschmidt, *Automorphisms of trivalent graphs*, Ann. of Math. **111** (1980) 377–406.
https://doi.org/10.2307/1971203

[11] H. Han and Z. Lu, *Semisymmetric graphs of order* $6p^2$ *and prime valency*, Sci. China Math. **55** (2012) 2579–2592.
https://doi.org/10.1007/s11425-012-4424-9

[12] M. Herzog, *On finite simple groups of order divisible by three primes only*, J. Algebra **120** (1968) 383–388.
https://doi.org/10.1016/0021-8693(68)90088-4

[13] X. Hua and Y. Feng, *Cubic semisymmetric graphs of order* $8p^3$, Sci. China Math. **54** (2011) 1937–1949.
https://doi.org/10.1007/s11425-011-4261-2

[14] J.H. Kwak and R. Nedela, Graphs and their Coverings (Lecture Notes Series No. 17, Combinatorial and Computational Mathematics Center, POSTECH, Pohang, Korea, 2005).

[15] Z. Lu, C. Wang and M. Xu, *On semisymmetric cubic graphs of order* $6p^2$, Sci. China Ser. A Math. **47** (2004) 1–17.
https://doi.org/10.1360/02ys0241

[16] A. Malnič, D. Marušič and C. Wang, Cubic Semisymmetric Graphs of Order $2p^3$ (University of Ljubljana, Preprint Series, Vol. 38, 2000).

[17] A. Malnič, D. Marušič and C. Wang, *Cubic edge-transitive graphs of order* $2p^3$, Discrete Math. **274** (2004) 187–198.
https://doi.org/10.1016/S0012-365X(03)00088-8

[18] D.J. Robinson, A Course in the Theory of Groups (Springer-Verlag, New York, 1982).
https://doi.org/10.1007/978-1-4684-0128-8

[19] J.S. Rose, A Course On Group Theory (Cambridge University Press, 1978).

[20] W.J. Shi, *On simple K_4-groups*, Chinese Sci. Bull. **36** (1991) 1281–1283.

[21] M. Suzuki, Group Theory (Springer-Verlag, New York, 1986).

[22] A.A. Talebi and N. Mehdipoor, *Classifying cubic semisymmetric graphs of order $18p^n$*, Graphs Combin. **30** (2014) 1037–1044.
https://doi.org/10.1007/s00373-013-1318-8

[23] W.T. Tutte, Connectivity in Graphs (University of Toronto Press, Toronto, 1966).

[24] C.Q. Wang and T.S. Chen, *Semisymmetric cubic graphs as regular covers of $K_{3,3}$*, Acta Math. Sin. (Engl. Ser.) **24** (2008) 405–416.
https://doi.org/10.1007/s10114-007-0998-5

[25] S. Zhang and W.J. Shi, *Revisiting the number of simple K_4-groups* (2013).
arXiv: 1307.8079v1 [math.NT]

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