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CONFLICT-FREE VERTEX CONNECTION NUMBER AT MOST 3 AND SIZE OF GRAPHS

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Abstract

A path in a vertex-coloured graph is called *conflict-free* if there is a colour used on exactly one of its vertices. A vertex-coloured graph is said to be *conflict-free vertex-connected* if any two distinct vertices of the graph are connected by a conflict-free vertex-path. The *conflict-free vertex-connection number*, denoted by $vcfc(G)$, is the smallest number of colours needed in order to make G conflict-free vertex-connected. Clearly, $vcfc(G) \geq 2$ for every connected graph on $n \geq 2$ vertices.

Our main result of this paper is the following. Let G be a connected graph of order n. If $|E(G)| \geq \binom{n-6}{2} + 7$, then $vcfc(G) \leq 3$. We also show that $vcfc(G) \leq k + 3 - t$ for every connected graph G with k cut-vertices and t being the maximum number of cut-vertices belonging to a block of G.

Keywords: vertex-colouring, conflict-free vertex-connection number, size of graph.

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