

## ON $(p, 1)$ -TOTAL LABELLING OF SOME 1-PLANAR GRAPHS<sup>1</sup>

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### Abstract

A graph is 1-planar if it can be drawn on the plane so that each edge is crossed by at most one other edge. In this paper, it is proved that the  $(p, 1)$ -total labelling number ( $p \geq 2$ ) of every 1-planar graph  $G$  is at most  $\Delta(G) + 2p - 2$  provided that  $\Delta(G) \geq 6p + 7$  or  $\Delta(G) \geq 4p + 6$  and  $G$  is triangle-free.

**Keywords:** 1-planar graph, total coloring,  $(p, 1)$ -total labelling, structural theorem.

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