Discussiones Mathematicae
Graph Theory 41 (2021) 481–501
doi:10.7151/dmgt.2205

INTERNALLY 4-CONNECTED GRAPHS WITH NO {CUBE, V_8}-MINOR

Chanun Lewchalermvongs

Department of Mathematics, Faculty of Science, and
Centre of Excellence in Mathematics
Mahidol University, Bangkok 10400, Thailand

e-mail: chanun.lew@mahidol.edu

AND

Nawarat Ananchuen

Centre of Excellence in Mathematics
Mahidol University, Bangkok 10400, Thailand

e-mail: nananchuen@yahoo.com

Abstract

A simple graph is a minor of another if the first is obtained from the second by deleting vertices, deleting edges, contracting edges, and deleting loops and parallel edges that are created when we contract edges. A cube is an internally 4-connected planar graph with eight vertices and twelve edges corresponding to the skeleton of the cube in the platonic solid, and the Wagner graph V_8 is an internally 4-connected nonplanar graph obtained from a cube by introducing a twist. A complete characterization of all internally 4-connected graphs with no V_8 minor is given in J. Maharry and N. Robertson, *The structure of graphs not topologically containing the Wagner graph*, J. Combin. Theory Ser. B 121 (2016) 398–420; on the other hand, only a characterization of 3-connected graphs with no cube minor is given in J. Maharry, *A characterization of graphs with no cube minor*, J. Combin. Theory Ser. B 80 (2008) 179–201. In this paper we determine all internally 4-connected graphs that contain neither cube nor V_8 as minors. This result provides a step closer to a complete characterization of all internally 4-connected graphs with no cube minor.

Keywords: internally 4-connected, minor, cube graph, V_8 graph.

2010 Mathematics Subject Classification: 05C83.

References

[1] C. Chun, D. Mayhew and J. Oxley, *Constructing internally 4-connected binary matroids*, Adv. Appl. Math. **50** (2013) 16–45.
doi:10.1016/j.aam.2012.03.005

[2] R. Diestel, Graph Theory (Springer Heidelberg Dordrecht London New York, 2010).

[3] G. Ding, *A characterization of graphs with no octahedron minor*, J. Graph Theory **74** (2013) 143–162.
doi:10.1002/jgt.21699

[4] G. Ding and C. Liu, *Excluding a small minor*, Discrete Appl. Math. **161** (2013) 355–368.
doi:10.1016/j.dam.2012.09.001

[5] G. Ding, C. Lewchalermvongs and J. Maharry, *Graphs with no $\bar{P}_7$-minor*, Electron. J. Combin. **23** (2016) #P2.16.

[6] J. Maharry, *An excluded minor theorem for the octahedron*, J. Graph Theory **31** (1999) 95–100.
doi:10.1002/(SICI)1097-0118(199906)31:2⟨95::AID-JGT2⟩3.0.CO;2-N

[7] J. Maharry, *A characterization of graphs with no cube minor*, J. Combin. Theory Ser. B **80** (2008) 179–201.
doi:10.1006/jctb.2000.1968

[8] J. Maharry, *An excluded minor theorem for the octahedron plus an edge*, J. Graph Theory **57** (2008) 124–130.
doi:10.1002/jgt.20272

[9] J. Maharry and N. Robertson, *The structure of graphs not topologically containing the Wagner graph*, J. Combin. Theory Ser. B **121** (2016) 398–420.
doi:10.1016/j.jctb.2016.07.011

Received 28 June 2018
Revised 29 December 2018
Accepted 10 January 2019