

CRITICAL AND FLOW-CRITICAL SNARKS COINCIDE

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Abstract

Over the past twenty years, critical and bicritical snarks have been appearing in the literature in various forms and in different contexts. Two main variants of criticality of snarks have been studied: criticality with respect to the non-existence of a 3-edge-colouring and criticality with respect to the non-existence of a nowhere-zero 4-flow. In this paper we show that these two kinds of criticality coincide, thereby completing previous partial results of de Freitas *et al.* [Electron. Notes Discrete Math. 50 (2015) 199–204] and Fiol *et al.* [Electron. J. Combin. 25 (2017) #P4.54].

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