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A NOTE ON THE CROSSING NUMBERS OF 5-REGULAR GRAPHS [1]

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Abstract

The crossing number $\mathrm{cr}(G)$ of a graph G is the smallest number of edge crossings in any drawing of G. In this paper, we prove that there exists a unique 5-regular graph G on 10 vertices with $\mathrm{cr}(G) = 2$. This answers a question by Chia and Gan in the negative. In addition, we also give a new proof of Chia and Gan's result which states that if G is a non-planar 5-regular graph on 12 vertices, then $\mathrm{cr}(G) \geq 2$.

Keywords: crossing number, 5-regular graph, drawing.

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