

***H*-KERNELS IN UNIONS OF *H*-COLORED QUASI-TRANSITIVE DIGRAPHS**

JOSÉ MANUEL CAMPERO-ALONZO¹

AND

ROCÍO SÁNCHEZ-LÓPEZ

Facultad de Ciencias
Universidad Nacional Autónoma de México
Círculo Exterior s/n, Coyoacán
Ciudad Universitaria, 04510, Ciudad de México, CDMX

e-mail: (Campero-Alonzo) cloud_1993@hotmail.com
(Sánchez-López) usagitsukinomx@yahoo.com.mx

Abstract

Let H be a digraph (possibly with loops) and D a digraph without loops whose arcs are colored with the vertices of H (D is said to be an H -colored digraph). For an arc (x, y) of D , its color is denoted by $c(x, y)$. A directed path $W = (v_0, \dots, v_n)$ in an H -colored digraph D will be called H -path if and only if $(c(v_0, v_1), \dots, c(v_{n-1}, v_n))$ is a directed walk in H . In W , we will say that there is an obstruction on v_i if $(c(v_{i-1}, v_i), c(v_i, v_{i+1})) \notin A(H)$ (if $v_0 = v_n$ we will take indices modulo n). A subset N of $V(D)$ is said to be an H -kernel in D if for every pair of different vertices in N there is no H -path between them, and for every vertex u in $V(D) \setminus N$ there exists an H -path in D from u to N . Let D be an arc-colored digraph. The color-class digraph of D , $\mathcal{C}_C(D)$, is the digraph such that $V(\mathcal{C}_C(D)) = \{c(a) : a \in A(D)\}$ and $(i, j) \in A(\mathcal{C}_C(D))$ if and only if there exist two arcs, namely (u, v) and (v, w) in D , such that $c(u, v) = i$ and $c(v, w) = j$. The main result establishes that if $D = D_1 \cup D_2$ is an H -colored digraph which is a union of asymmetric quasi-transitive digraphs and $\{V_1, \dots, V_k\}$ is a partition of $V(\mathcal{C}_C(D))$ with a property P^* such that

1. V_i is a quasi-transitive V_i -class for every i in $\{1, \dots, k\}$,
2. either $D[\{a \in A(D) : c(a) \in V_i\}]$ is a subdigraph of D_1 or it is a subdigraph of D_2 for every i in $\{1, \dots, k\}$,
3. D_i has no infinite outward path for every i in $\{1, 2\}$,

¹Corresponding author.

4. every cycle of length three in D has at most two obstructions,
then D has an H -kernel.

Some results with respect to the existence of kernels by monochromatic paths in finite digraphs will be deduced from the main result.

Keywords: quasi-transitive digraph, kernel by monochromatic paths, alternating kernel, H -kernel, obstruction.

2010 Mathematics Subject Classification: 05C20, 05C38, 05C69.

REFERENCES

- [1] P. Arpin and V. Linek, *Reachability problems in edge-colored digraphs*, Discrete Math. **307** (2007) 2276–2289.
doi:10.1016/j.disc.2006.09.042
- [2] J. Bang-Jensen and G. Gutin, *Digraphs: Theory, Algorithms and Applications* (Springer, London, 2009).
doi:10.1007/978-1-84800-998-1
- [3] Y. Bai, S. Fujita and S. Zhang, *Kernels by properly colored paths in arc-colored digraphs*, Discrete Math. **341** (2018) 1523–1533.
doi:10.1016/j.disc.2018.02.014
- [4] C. Berge, *Graphs* (North-Holland, Amsterdam, 1989).
- [5] C. Berge and P. Duchet, *Recent problems and results about kernels in directed graphs*, Discrete Math. **86** (1990) 27–31.
doi:10.1016/0012-365X(90)90346-J
- [6] P. Delgado-Escalante and H. Galeana-Sánchez, *Restricted domination in arc-colored digraphs*, AKCE Int. J. Graphs Comb. **11** (2014) 95–104.
- [7] P. Delgado-Escalante, H. Galeana-Sánchez and E. O'Reilly-Regueiro, *Alternating kernels*, Discrete Appl. Math. **236** (2018) 153–164.
doi:10.1016/j.dam.2017.10.013
- [8] H. Galeana-Sánchez, *Kernels by monochromatic paths and the color-class digraph*, Discuss. Math. Graph Theory **31** (2011) 273–281.
doi:10.7151/dmgt.1544
- [9] H. Galeana-Sánchez, *On monochromatic paths and monochromatic cycles in edge coloured tournaments*, Discrete Math. **156** (1996) 103–112.
doi:10.1016/0012-365X(95)00036-V
- [10] H. Galeana-Sánchez and R. Rojas-Monroy, *Kernels in quasi-transitive digraphs*, Discrete Math. **306** (2006) 1969–1974.
doi:10.1016/j.disc.2006.02.015
- [11] H. Galeana-Sánchez, B. Llano and J.J. Montellano-Ballesteros, *Kernels by monochromatic paths in m -colored unions of quasi-transitive digraphs*, Discrete Appl. Math. **158** (2010) 461–466.
doi:10.1016/j.dam.2009.11.005

- [12] A. Ghouilá-Houri, *Caractérisation des graphes non orientés dont on peut orienter les arêtes de manière à obtenir le graphe d'une relation d'ordre*, C. R. Math. Acad. Sci. Paris **254** (1962) 1370–1371.
- [13] V. Linek and B. Sands, *A note on paths in edge-colored tournaments*, Ars Combin. **44** (1996) 225–228.
- [14] K.B. Reid, *Monotone reachability in arc-colored tournaments*, Congr. Numer. **146** (2000) 131–141.
- [15] B. Sands, N. Sauer and R. Woodrow, *On monochromatic paths in edge-coloured digraphs*, J. Combin. Theory Ser. B **33** (1982) 271–275.
doi:10.1016/0095-8956(82)90047-8

Received 14 March 2018
 Revised 6 November 2018
 Accepted 7 November 2018