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H-KERNELS IN UNIONS OF *H*-COLORED QUASI-TRANSITIVE DIGRAPHS

José Manuel Campero-Alonzo[1]

and

Rocío Sánchez-López

Facultad de Ciencias
Universidad Nacional Autónoma de México
Círcuito Exterior s/n, Coyoacán
Ciudad Universitaria, 04510, Ciudad de México, CDMX

e-mail: (Campero-Alonzo) cloud_1993@hotmail.com
(Sánchez-López) usagitsukinomx@yahoo.com.mx

Abstract

Let H be a digraph (possibly with loops) and D a digraph without loops whose arcs are colored with the vertices of H (D is said to be an H-colored digraph). For an arc (x, y) of D, its color is denoted by $c(x, y)$. A directed path $W = (v_0, \ldots, v_n)$ in an H-colored digraph D will be called H-path if and only if $(c(v_0, v_1), \ldots, c(v_{n-1}, v_n))$ is a directed walk in H. In W, we will say that there is an obstruction on v_i if $(c(v_{i-1}, v_i), c(v_i, v_{i+1})) \notin A(H)$ (if $v_0 = v_n$ we will take indices modulo n). A subset N of $V(D)$ is said to be an H-kernel in D if for every pair of different vertices in N there is no H-path between them, and for every vertex u in $V(D) \setminus N$ there exists an H-path in D from u to N. Let D be an arc-colored digraph. The color-class digraph of D, $\mathscr{C}_C(D)$, is the digraph such that $V(\mathscr{C}_C(D)) = \{c(a) : a \in A(D)\}$ and $(i, j) \in A(\mathscr{C}_C(D))$ if and only if there exist two arcs, namely (u, v) and (v, w) in D, such that $c(u, v) = i$ and $c(v, w) = j$. The main result establishes that if $D = D_1 \cup D_2$ is an H-colored digraph which is a union of asymmetric quasi-transitive digraphs and $\{V_1, \ldots, V_k\}$ is a partition of $V(\mathscr{C}_C(D))$ with a property P^* such that

1. V_i is a quasi-transitive V_i-class for every i in $\{1, \ldots, k\}$,
2. either $D[\{a \in A(D) : c(a) \in V_i\}]$ is a subdigraph of D_1 or it is a sudigraph of D_2 for every i in $\{1, \ldots, k\}$,
3. D_i has no infinite outward path for every i in $\{1, 2\}$,

[1]Corresponding author.

4. every cycle of length three in D has at most two obstructions,

then D has an H-kernel.

Some results with respect to the existence of kernels by monochromatic paths in finite digraphs will be deduced from the main result.

Keywords: quasi-transitive digraph, kernel by monochromatic paths, alternating kernel, H-kernel, obstruction.

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