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T-COLORINGS, DIVISIBILITY AND THE CIRCULAR CHROMATIC NUMBER

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Abstract

Let T be a T-set, i.e., a finite set of nonnegative integers satisfying $0 \in T$, and G be a graph. In the paper we study relations between the T-edge spans $\mathrm{esp}_T(G)$ and $\mathrm{esp}_{d\odot T}(G)$, where d is a positive integer and

$$d \odot T = \{0 \le t \le d\,(\max T + 1) \colon d \mid t \Rightarrow t/d \in T\}\,.$$

We show that $\mathrm{esp}_{d\odot T}(G) = d\,\mathrm{esp}_T(G) - r$, where r, $0 \le r \le d-1$, is an integer that depends on T and G. Next we focus on the case $T = \{0\}$ and show that

$$\mathrm{esp}_{d\odot\{0\}}(G) = \lceil d(\chi_c(G) - 1)\rceil,$$

where $\chi_c(G)$ is the circular chromatic number of G. This result allows us to formulate several interesting conclusions that include a new formula for the circular chromatic number

$$\chi_c(G) = 1 + \inf\left\{\mathrm{esp}_{d\odot\{0\}}(G)/d \colon d \ge 1\right\}$$

and a proof that the formula for the T-edge span of powers of cycles, stated as conjecture in [Y. Zhao, W. He and R. Cao, *The edge span of T-coloring on graph C_n^d*, Appl. Math. Lett. 19 (2006) 647–651], is true.

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