

***T*-COLORINGS, DIVISIBILITY AND THE CIRCULAR CHROMATIC NUMBER**

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Abstract

Let T be a T -set, i.e., a finite set of nonnegative integers satisfying $0 \in T$, and G be a graph. In the paper we study relations between the T -edge spans $\text{esp}_T(G)$ and $\text{esp}_{d \odot T}(G)$, where d is a positive integer and

$$d \odot T = \{0 \leq t \leq d(\max T + 1) : d \mid t \Rightarrow t/d \in T\}.$$

We show that $\text{esp}_{d \odot T}(G) = d \text{esp}_T(G) - r$, where r , $0 \leq r \leq d - 1$, is an integer that depends on T and G . Next we focus on the case $T = \{0\}$ and show that

$$\text{esp}_{d \odot \{0\}}(G) = \lceil d(\chi_c(G) - 1) \rceil,$$

where $\chi_c(G)$ is the circular chromatic number of G . This result allows us to formulate several interesting conclusions that include a new formula for the circular chromatic number

$$\chi_c(G) = 1 + \inf \{ \text{esp}_{d \odot \{0\}}(G)/d : d \geq 1 \}$$

and a proof that the formula for the T -edge span of powers of cycles, stated as conjecture in [Y. Zhao, W. He and R. Cao, *The edge span of T -coloring on graph C_n^d* , Appl. Math. Lett. 19 (2006) 647–651], is true.

Keywords: T -coloring, circular chromatic number.

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