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DECOMPOSITIONS OF COMPLETE BIPARTITE GRAPHS AND COMPLETE GRAPHS INTO PATHS, STARS, AND CYCLES WITH FOUR EDGES EACH

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Abstract

Let G be either a complete graph of odd order or a complete bipartite graph in which each vertex partition has an even number of vertices. In this paper, we determine the set of triples (p, q, r), with p, q, $r > 0$, for which there exists a decomposition of G into p paths, q stars, and r cycles, each of which has 4 edges.

Keywords: complete graph, complete bipartite graph, path, star, cycle, decomposition.

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