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ON THE STAR CHROMATIC INDEX OF GENERALIZED PETERSEN GRAPHS

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Abstract

The star k-edge-coloring of graph G is a proper edge coloring using k colors such that no path or cycle of length four is bichromatic. The minimum number k for which G admits a star k-edge-coloring is called the star chromatic index of G, denoted by $\chi'_s(G)$. Let $\mathrm{GCD}(n,k)$ be the greatest common divisor of n and k. In this paper, we give a necessary and sufficient condition of $\chi'_s(P(n,k)) = 4$ for a generalized Petersen graph $P(n,k)$ and show that "almost all" generalized Petersen graphs have a star 5-edge-colorings. Furthermore, for any two integers k and n ($\geq 2k+1$) such that $\mathrm{GCD}(n,k) \geq 3$, $P(n,k)$ has a star 5-edge-coloring, with the exception of the case that $\mathrm{GCD}(n,k) = 3$, $k \neq \mathrm{GCD}(n,k)$ and $\frac{n}{3} \equiv 1 \pmod 3$.

Keywords: star edge-coloring, star chromatic index, generalized Petersen graph.

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