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COVERINGS OF CUBIC GRAPHS AND 3-EDGE COLORABILITY

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Abstract

Let $h\colon \tilde{G} \to G$ be a finite covering of 2-connected cubic (multi)graphs where G is 3-edge uncolorable. In this paper, we describe conditions under which $\tilde{G}$ is 3-edge uncolorable. As particular cases, we have constructed regular and irregular 5-fold coverings $f\colon \tilde{G} \to G$ of uncolorable cyclically 4-edge connected cubic graphs and an irregular 5-fold covering $g\colon \tilde{H} \to H$ of uncolorable cyclically 6-edge connected cubic graphs.

In [13], Steffen introduced the resistance of a subcubic graph, a characteristic that measures how far is this graph from being 3-edge colorable. In this paper, we also study the relation between the resistance of the base cubic graph and the covering cubic graph.

Keywords: uncolorable cubic graph, covering of graphs, voltage permutation graph, resistance, nowhere-zero 4-flow.

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