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TOTAL 2-RAINBOW DOMINATION NUMBERS OF TREES

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Abstract

A 2-rainbow dominating function (2RDF) of a graph $G = (V(G), E(G))$ is a function f from the vertex set $V(G)$ to the set of all subsets of the set $\{1, 2\}$ such that for every vertex $v \in V(G)$ with $f(v) = \emptyset$ the condition $\bigcup_{u \in N(v)} f(u) = \{1, 2\}$ is fulfilled, where $N(v)$ is the open neighborhood of v. A total 2-rainbow dominating function f of a graph with no isolated vertices is a 2RDF with the additional condition that the subgraph of G induced by $\{v \in V(G) \mid f(v) \neq \emptyset\}$ has no isolated vertex. The total 2-rainbow domination number, $\gamma_{tr2}(G)$, is the minimum weight of a total 2-rainbow dominating function of G. In this paper, we establish some sharp upper and lower bounds on the total 2-rainbow domination number of a tree. Moreover, we show that the decision problem associated with $\gamma_{tr2}(G)$ is NP-complete for bipartite and chordal graphs.

Keywords: 2-rainbow dominating function, 2-rainbow domination number, total 2-rainbow dominating function, total 2-rainbow domination number.

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