

## STAR-CRITICAL RAMSEY NUMBERS FOR CYCLES VERSUS $K_4$

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### Abstract

Given three graphs  $G$ ,  $H$  and  $K$  we write  $K \rightarrow (G, H)$ , if in any red/blue coloring of the edges of  $K$  there exists a red copy of  $G$  or a blue copy of  $H$ . The Ramsey number  $r(G, H)$  is defined as the smallest natural number  $n$  such that  $K_n \rightarrow (G, H)$  and the star-critical Ramsey number  $r_*(G, H)$  is defined as the smallest positive integer  $k$  such that  $K_{n-1} \sqcup K_{1,k} \rightarrow (G, H)$ , where  $n$  is the Ramsey number  $r(G, H)$ . When  $n \geq 3$ , we show that  $r_*(C_n, K_4) = 2n$  except for  $r_*(C_3, K_4) = 8$  and  $r_*(C_4, K_4) = 9$ . We also characterize all Ramsey critical  $r(C_n, K_4)$  graphs.

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