

STAR-CRITICAL RAMSEY NUMBERS FOR CYCLES VERSUS K_4

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Abstract

Given three graphs G , H and K we write $K \rightarrow (G, H)$, if in any red/blue coloring of the edges of K there exists a red copy of G or a blue copy of H . The Ramsey number $r(G, H)$ is defined as the smallest natural number n such that $K_n \rightarrow (G, H)$ and the star-critical Ramsey number $r_*(G, H)$ is defined as the smallest positive integer k such that $K_{n-1} \sqcup K_{1,k} \rightarrow (G, H)$, where n is the Ramsey number $r(G, H)$. When $n \geq 3$, we show that $r_*(C_n, K_4) = 2n$ except for $r_*(C_3, K_4) = 8$ and $r_*(C_4, K_4) = 9$. We also characterize all Ramsey critical $r(C_n, K_4)$ graphs.

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