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CHANGING AND UNCHANGING OF THE DOMINATION NUMBER OF A GRAPH: PATH ADDITION NUMBERS

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Abstract

Given a graph $G = (V, E)$ and two its distinct vertices u and v, the (u, v)-P_k-*addition graph* of G is the graph $G_{u,v,k-2}$ obtained from disjoint union of G and a path $P_k : x_0, x_1, \ldots, x_{k-1}$, $k \geq 2$, by identifying the vertices u and x_0, and identifying the vertices v and x_{k-1}. We prove that $\gamma(G) - 1 \leq \gamma(G_{u,v,k})$ for all $k \geq 1$, and $\gamma(G_{u,v,k}) > \gamma(G)$ when $k \geq 5$. We also provide necessary and sufficient conditions for the equality $\gamma(G_{u,v,k}) = \gamma(G)$ to be valid for each pair $u, v \in V(G)$. In addition, we establish sharp upper and lower bounds for the minimum, respectively maximum, k in a graph G over all pairs of vertices u and v in G such that the (u, v)-P_k-addition graph of G has a larger domination number than G, which we consider separately for adjacent and non-adjacent pairs of vertices.

Keywords: domination number, path addition.

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