

## CHANGING AND UNCHANGING OF THE DOMINATION NUMBER OF A GRAPH: PATH ADDITION NUMBERS

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### Abstract

Given a graph  $G = (V, E)$  and two its distinct vertices  $u$  and  $v$ , the  $(u, v)$ - $P_k$ -addition graph of  $G$  is the graph  $G_{u,v,k-2}$  obtained from disjoint union of  $G$  and a path  $P_k : x_0, x_1, \dots, x_{k-1}$ ,  $k \geq 2$ , by identifying the vertices  $u$  and  $x_0$ , and identifying the vertices  $v$  and  $x_{k-1}$ . We prove that  $\gamma(G) - 1 \leq \gamma(G_{u,v,k})$  for all  $k \geq 1$ , and  $\gamma(G_{u,v,k}) > \gamma(G)$  when  $k \geq 5$ . We also provide necessary and sufficient conditions for the equality  $\gamma(G_{u,v,k}) = \gamma(G)$  to be valid for each pair  $u, v \in V(G)$ . In addition, we establish sharp upper and lower bounds for the minimum, respectively maximum,  $k$  in a graph  $G$  over all pairs of vertices  $u$  and  $v$  in  $G$  such that the  $(u, v)$ - $P_k$ -addition graph of  $G$  has a larger domination number than  $G$ , which we consider separately for adjacent and non-adjacent pairs of vertices.

**Keywords:** domination number, path addition.

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