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ON THE RESTRICTED SIZE RAMSEY NUMBER INVOLVING A PATH P_3

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Abstract

For any pair of graphs G and H, both the size Ramsey number $\hat{r}(G,H)$ and the restricted size Ramsey number $r^*(G,H)$ are bounded above by the size of the complete graph with order equals to the Ramsey number $r(G,H)$, and bounded below by $e(G)+e(H)-1$. Moreover, trivially, $\hat{r}(G,H) \leq r^*(G,H)$. When introducing the size Ramsey number for graph, Erdős *et al.* (1978) asked two questions; (1) Do there exist graphs G and H such that $\hat{r}(G,H)$ attains the upper bound? and (2) Do there exist graphs G and H such that $\hat{r}(G,H)$ is significantly less than the upper bound?

In this paper we consider the restricted size Ramsey number $r^*(G,H)$. We answer both questions above for $r^*(G,H)$ when $G = P_3$ and H is a connected graph.

Keywords: restricted size Ramsey number, path, connected graph, star.
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