

NOTE

GRAPH EXPONENTIATION AND NEIGHBORHOOD RECONSTRUCTION

RICHARD H. HAMMACK¹

*Department of Mathematics and Applied Mathematics
Virginia Commonwealth University
Richmond, VA 23284-2014, USA*

e-mail: rhammack@vcu.edu

Abstract

Any graph G admits a neighborhood multiset $\mathcal{N}(G) = \{N_G(x) \mid x \in V(G)\}$ whose elements are precisely the open neighborhoods of G . We say G is neighborhood reconstructible if it can be reconstructed from $\mathcal{N}(G)$, that is, if $G \cong H$ whenever $\mathcal{N}(G) = \mathcal{N}(H)$ for some other graph H . This note characterizes neighborhood reconstructible graphs as those graphs G that obey the exponential cancellation $G^{K_2} \cong H^{K_2} \implies G \cong H$.

Keywords: neighborhood reconstructible graphs, graph exponentiation.

2010 Mathematics Subject Classification: 05C60, 05C76.

REFERENCES

- [1] M. Aigner and E. Triesch, *Realizability and uniqueness in graphs*, Discrete Math. **136** (1994) 3–20.
doi:10.1016/0012-365X(94)00104-Q
- [2] R. Hammack, W. Imrich and S. Klavžar, Handbook of Product Graphs, Second Edition, Series: Discrete Mathematics and its Applications (CRC Press, 2011).
- [3] R. Hammack and C. Mullican, *Neighborhood reconstruction and cancellation of graphs*, Electron. J. Combin. **24** (2017) #P2.8
- [4] B. Jónsson, *Arithmetic of ordered sets*, in: I. Rival (Ed.), Ordered Sets (Banff Alta, 1981), Volume 83 of NATO Advanced Study Inst. Ser. C. Math. Phys. Sci. 3–43.
- [5] R. Mizzi, Two-Fold Isomorphisms, Ph.D. Dissertation (University of Malta, 2016).

¹Supported by Simons Collaboration Grant for Mathematicians 523748.

- [6] L. Lovász, *Operations with structures*, Acta Math. Acad. Sci. Hungar. **18** (1967) 321–328.
doi:10.1007/BF02280291
- [7] L. Lovász, *On the cancellation law among finite relational structures*, Period. Math. Hungar. **1** (1971) 145–156.
doi:10.1007/BF02029172
- [8] N. Sauer, *Hedetniemi’s conjecture—a survey*, Discrete Math. **229** (2001) 261–292.
doi:10.1016/S0012-365X(00)00213-2
- [9] V.T. Sós, *Problem*, in: A. Hajnal and V.T. Sós, (Eds.), Combinatorics Coll. Math. Soc. J. Bolyai 18 (North-Holland, Amsterdam, 1978) 1214.

Received 3 September 2018

Accepted 27 October 2018