

ECCENTRICITY OF NETWORKS WITH STRUCTURAL CONSTRAINTS ¹

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Abstract

The eccentricity of a node v in a network is the maximum distance from v to any other node. In social networks, the reciprocal of eccentricity is used as a measure of the importance of a node within a network. The associated centralization measure then calculates the degree to which a network is dominated by a particular node. In this work, we determine the maximum value of eccentricity centralization as well as the most centralized networks for various classes of networks including the families of bipartite networks (two-mode data) with given partition sizes and tree networks with fixed number of nodes and fixed maximum degree. To this end, we introduce and study a new way of enumerating the nodes of a tree which might be of independent interest.

Keywords: eccentricity, network, bipartite graph, complex network, maximum degree.

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