Discussiones Mathematicae
Graph Theory 41 (2021) 249–266
doi:10.7151/dmgt.2178

DECOMPOSITION OF THE TENSOR PRODUCT OF COMPLETE GRAPHS INTO CYCLES OF LENGTHS 3 AND 6

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Abstract

By a $\{C_3^\alpha, C_6^\beta\}$-decomposition of a graph G, we mean a partition of the edge set of G into α cycles of length 3 and β cycles of length 6. In this paper, necessary and sufficient conditions for the existence of a $\{C_3^\alpha, C_6^\beta\}$-decomposition of $(K_m \times K_n)(\lambda)$, where $\times$ denotes the tensor product of graphs and λ is the multiplicity of the edges, is obtained. In fact, we prove that for $\lambda \geq 1$, $m, n \geq 3$ and $(m, n) \neq (3, 3)$, a $\{C_3^\alpha, C_6^\beta\}$-decomposition of $(K_m \times K_n)(\lambda)$ exists if and only if $\lambda(m-1)(n-1) \equiv 0 \pmod 2$ and $3\alpha + 6\beta = \frac{\lambda m(m-1)n(n-1)}{2}$.

Keywords: cycle decomposition, tensor product.

2010 Mathematics Subject Classification: 05B30, 05C70.

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Received 3 March 2018
Revised 3 October 2018
Accepted 3 October 2018