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DOMINATING VERTEX COVERS: THE VERTEX-EDGE DOMINATION PROBLEM

William F. Klostermeyer

School of Computing
University of North Florida
Jacksonville, FL 32224 USA

e-mail: wkloster@unf.edu

Margaret-Ellen Messinger

Department of Mathematics and Computer Science
Mount Allison University, Sackville, NB, Canada

e-mail: mmessinger@mta.ca

and

Anders Yeo

Department of Mathematics and Computer Science
University of Southern Denmark
Odense, Denmark
and Department of Mathematics
University of Johannesburg
Auckland Park, 2006 South Africa

e-mail: yeo@imada.sdu.dk

Abstract

The *vertex-edge domination number* of a graph, $\gamma_{ve}(G)$, is defined to be the cardinality of a smallest set D such that there exists a vertex cover C of G such that each vertex in C is dominated by a vertex in D. This is motivated by the problem of determining how many guards are needed in a graph so that a searchlight can be shone down each edge by a guard either incident to that edge or at most distance one from a vertex incident to the edge. Our main result is that for any cubic graph G with n vertices, $\gamma_{ve}(G) \leq 9n/26$. We also show that it is NP-hard to decide if $\gamma_{ve}(G) = \gamma(G)$ for bipartite graph G.

Keywords: cubic graph, dominating set, vertex cover, vertex-edge dominating set.

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