

DOMINATING VERTEX COVERS: THE VERTEX-EDGE DOMINATION PROBLEM

WILLIAM F. KLOSTERMEYER

*School of Computing
University of North Florida
Jacksonville, FL 32224 USA*

e-mail: wkloster@unf.edu

MARGARET-ELLEN MESSINGER

*Department of Mathematics and Computer Science
Mount Allison University, Sackville, NB, Canada*

e-mail: mmessinger@mta.ca

AND

ANDERS YEO

*Department of Mathematics and Computer Science
University of Southern Denmark
Odense, Denmark
and Department of Mathematics
University of Johannesburg
Auckland Park, 2006 South Africa*

e-mail: yeo@imada.sdu.dk

Abstract

The *vertex-edge domination number* of a graph, $\gamma_{ve}(G)$, is defined to be the cardinality of a smallest set D such that there exists a vertex cover C of G such that each vertex in C is dominated by a vertex in D . This is motivated by the problem of determining how many guards are needed in a graph so that a searchlight can be shone down each edge by a guard either incident to that edge or at most distance one from a vertex incident to the edge. Our main result is that for any cubic graph G with n vertices, $\gamma_{ve}(G) \leq 9n/26$. We also show that it is *NP*-hard to decide if $\gamma_{ve}(G) = \gamma(G)$ for bipartite graph G .

Keywords: cubic graph, dominating set, vertex cover, vertex-edge dominating set.

2010 Mathematics Subject Classification: 05C69.

REFERENCES

- [1] R. Boutrig, M. Chellali, T.W. Haynes and S.T. Hedetniemi, *Vertex-edge domination in graphs*, Aequationes Math. **90** (2016) 355–366.
doi:10.1007/s00010-015-0354-2
- [2] D. Dereniowski, H. Ono, I. Suzuki, L. Wrona, M. Yamashita and P. Żyliński, *The searchlight problem for road networks*, Theoret. Comput. Sci. **591** (2015) 28–59.
doi:10.1016/j.tcs.2015.04.026
- [3] M.A. Henning and A. Yeo, *Transversals in 4-uniform hypergraphs*, Electron. J. Combin. **23** (2016) #P3.50.
- [4] W.F. Klostermeyer and C.M. Mynhardt, *Edge protection in graphs*, Australas. J. Combin. **45** (2009) 235–250.
- [5] W.F. Klostermeyer and C.M. Mynhardt, *Protecting a graph with mobile guards*, Appl. Anal. Discrete Math. **10** (2016) 1–29.
doi:10.2298/AADM151109021K
- [6] A.V. Kostochka and C. Stocker, *A new bound on the domination number of connected cubic graphs*, Sib. Èlektron. Mat. Izv. **6** (2009) 465–504.
- [7] B. Krishnakumari, Y.B. Venkatakrishnan and M. Krzywkowski, *Bounds on the vertex-edge domination number of a tree*, C.R. Math. **352** (2014) 363–366.
doi:10.1016/j.crma.2014.03.017
- [8] J.R. Lewis, Vertex-Edge and Edge-Vertex Domination in Graphs, Ph.D. Thesis (Clemson University, Clemson, 2007).
- [9] J.R. Lewis, S.T. Hedetniemi, T.W. Haynes and G.H. Fricke, *Vertex-edge domination*, Util. Math. **81** (2010) 193–213.
- [10] J.W. Peters, Theoretical and Algorithmic Results on Domination and Connectivity, Ph.D. Thesis (Clemson University, Clemson, 1986).
- [11] Y.B. Venkatakrishnan, C. Natarajan and G. Sathiamoorthy, *Vertex-edge and connected domination numbers of a tree*, Int. J. Pure Appl. Math. **119** (2018) 103–111.
- [12] W.C.K. Yen and C.Y. Tang, *An optimal algorithm for solving the searchlight guarding problem on weight two-terminal series-parallel graphs*, Acta Inform. **36** (1999) 143–172.
doi:10.1007/s002360050156

Received 4 January 2018

Revised 25 August 2018

Accepted 25 August 2018