

## GENERALIZED SUM LIST COLORINGS OF GRAPHS

ARNFRIED KEMNITZ

MASSIMILIANO MARANGIO

*Computational Mathematics*  
Technical University Braunschweig  
Universitätsplatz 2, 38106 Braunschweig, Germany

e-mail: a.kemnitz@tu-bs.de  
m.marangio@tu-bs.de

AND

MARGIT VOIGT

*Faculty of Information Technology and Mathematics*  
University of Applied Sciences  
Friedrich-List-Platz 1, 01069 Dresden, Germany  
e-mail: mvoigt@informatik.htw-dresden.de

### Abstract

A (graph) property  $\mathcal{P}$  is a class of simple finite graphs closed under isomorphisms. In this paper we consider generalizations of sum list colorings of graphs with respect to properties  $\mathcal{P}$ .

If to each vertex  $v$  of a graph  $G$  a list  $L(v)$  of colors is assigned, then in an  $(L, \mathcal{P})$ -coloring of  $G$  every vertex obtains a color from its list and the subgraphs of  $G$  induced by vertices of the same color are always in  $\mathcal{P}$ . The  $\mathcal{P}$ -sum choice number  $\chi_{sc}^{\mathcal{P}}(G)$  of  $G$  is the minimum of the sum of all list sizes such that, for any assignment  $L$  of lists of colors with the given sizes, there is always an  $(L, \mathcal{P})$ -coloring of  $G$ .

We state some basic results on monotonicity, give upper bounds on the  $\mathcal{P}$ -sum choice number of arbitrary graphs for several properties, and determine the  $\mathcal{P}$ -sum choice number of specific classes of graphs, namely, of all complete graphs, stars, paths, cycles, and all graphs of order at most 4.

**Keywords:** sum list coloring, sum choice number, generalized sum list coloring, additive hereditary graph property.

**2010 Mathematics Subject Classification:** 05C15.

## REFERENCES

- [1] A. Berliner, U. Bostelmann, R.A. Brualdi and L. Deaett, *Sum list coloring graphs*, Graphs Combin. **22** (2006) 173–183.  
doi:10.1007/s00373-005-0645-9
- [2] C. Brause, A. Kemnitz, M. Marangio, A. Pruchnewski and M. Voigt, *Sum choice number of generalized  $\theta$ -graphs*, Discrete Math. **340** (2017) 2633–2640.  
doi:10.1016/j.disc.2016.11.028
- [3] M. Borowiecki, I. Broere, M. Frick, P. Mihók and G. Semanišin, *A survey of hereditary properties of graphs*, Discuss. Math. Graph Theory **17** (1997) 5–50.  
doi:10.7151/dmgt.1037
- [4] E. Drgas-Burchardt and A. Drzystek, *General and acyclic sum-list-colouring of graphs*, Appl. Anal. Discrete Math. **10** (2016) 479–500.  
doi:10.2298/AADM161011026D
- [5] E. Drgas-Burchardt and A. Drzystek, *Acyclic sum-list-colouring of grids and other classes of graphs*, Opuscula Math. **37** (2017) 535–556.  
doi:10.7494/OpMath.2017.37.4.535
- [6] J. Harant and A. Kemnitz, *Lower bounds on the sum choice number of a graph*, Electron. Notes Discrete Math. **53** (2016) 421–431.  
doi:10.1016/j.endm.2016.05.036
- [7] G. Isaak, *Sum list coloring  $2 \times n$  arrays*, Electron. J. Combin. **9** (2002) #N8.
- [8] G. Isaak, *Sum list coloring block graphs*, Graphs Combin. **20** (2004) 499–506.  
doi:10.1007/s00373-004-0564-1
- [9] A. Kemnitz, M. Marangio and M. Voigt, *Bounds for the sum choice number*, Electron. Notes Discrete Math. **63** (2017) 49–58.  
doi:10.1016/j.endm.2017.10.061
- [10] A. Kemnitz, M. Marangio and M. Voigt, *On the  $\mathcal{P}$ -sum choice number of graphs for 1-additive properties*, Congr. Numer. **229** (2017) 117–124.
- [11] M.A. Lastrina, List-Coloring and Sum-List-Coloring Problems on Graphs, Ph.D. Thesis (Iowa State University, 2012).

Received 16 October 2017

Revised 5 August 2018

Accepted 6 September 2018