

## GENERALIZED SUM LIST COLORINGS OF GRAPHS

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### Abstract

A (graph) property  $\mathcal{P}$  is a class of simple finite graphs closed under isomorphisms. In this paper we consider generalizations of sum list colorings of graphs with respect to properties  $\mathcal{P}$ .

If to each vertex  $v$  of a graph  $G$  a list  $L(v)$  of colors is assigned, then in an  $(L, \mathcal{P})$ -coloring of  $G$  every vertex obtains a color from its list and the subgraphs of  $G$  induced by vertices of the same color are always in  $\mathcal{P}$ . The  $\mathcal{P}$ -sum choice number  $\chi_{sc}^{\mathcal{P}}(G)$  of  $G$  is the minimum of the sum of all list sizes such that, for any assignment  $L$  of lists of colors with the given sizes, there is always an  $(L, \mathcal{P})$ -coloring of  $G$ .

We state some basic results on monotonicity, give upper bounds on the  $\mathcal{P}$ -sum choice number of arbitrary graphs for several properties, and determine the  $\mathcal{P}$ -sum choice number of specific classes of graphs, namely, of all complete graphs, stars, paths, cycles, and all graphs of order at most 4.

**Keywords:** sum list coloring, sum choice number, generalized sum list coloring, additive hereditary graph property.

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