

## INTERSECTION DIMENSION AND GRAPH INVARIANTS

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### Abstract

We show that the intersection dimension of graphs with respect to several hereditary properties can be bounded as a function of the maximum degree. As an interesting special case, we show that the circular dimension of a graph with maximum degree  $\Delta$  is at most  $O\left(\Delta \frac{\log \Delta}{\log \log \Delta}\right)$ . It is also shown that permutation dimension of any graph is at most  $\Delta(\log \Delta)^{1+o(1)}$ . We also obtain bounds on intersection dimension in terms of treewidth.

**Keywords:** circular dimension, dimensional properties, forbidden-subgraph colorings.

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