

LARGE CONTRACTIBLE SUBGRAPHS OF A 3-CONNECTED GRAPH

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Abstract

Let $m \geq 5$ be a positive integer and let G be a 3-connected graph on at least $2m + 1$ vertices. We prove that G has a contractible set W such that $m \leq |W| \leq 2m - 4$. (Recall that a set $W \subset V(G)$ of a 3-connected graph G is contractible if the graph $G(W)$ is connected and the graph $G - W$ is 2-connected.) A particular case for $m = 4$ is that any 3-connected graph on at least 11 vertices has a contractible set of 5 or 6 vertices.

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