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ON IMPLICIT HEAVY SUBGRAPHS AND HAMILTONICITY OF 2-CONNECTED GRAPHS

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Abstract

A graph G of order n is *implicit claw-heavy* if in every induced copy of $K_{1,3}$ in G there are two non-adjacent vertices with sum of their implicit degrees at least n. We study various implicit degree conditions (including, but not limiting to, Ore- and Fan-type conditions) imposing of which on specific induced subgraphs of a 2-connected implicit claw-heavy graph ensures its Hamiltonicity. In particular, we improve a recent result of [X. Huang, *Implicit degree condition for Hamiltonicity of* 2*-heavy graphs*, Discrete Appl. Math. 219 (2017) 126–131] and complete the characterizations of pairs of o-heavy and f-heavy subgraphs for Hamiltonicity of 2-connected graphs.

Keywords: implicit degree, implicit o-heavy, implicit f-heavy, implicit c-heavy, Hamilton cycle.

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