Discussiones Mathematicae
Graph Theory 41 (2021) 51–64
doi:10.7151/dmgt.2169

LINEAR LIST COLORING OF SOME SPARSE GRAPHS

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Abstract

A linear k-coloring of a graph is a proper k-coloring of the graph such that any subgraph induced by the vertices of any pair of color classes is a union of vertex-disjoint paths. A graph G is linearly L-colorable if there is a linear coloring c of G for a given list assignment $L = \{L(v) : v \in V(G)\}$ such that $c(v) \in L(v)$ for all $v \in V(G)$, and G is linearly k-choosable if G is linearly L-colorable for any list assignment with $|L(v)| \geq k$. The smallest integer k such that G is linearly k-choosable is called the linear list chromatic number, denoted by $lc_l(G)$. It is clear that $lc_l(G) \geq \left\lceil \frac{\Delta(G)}{2} \right\rceil + 1$ for any graph G with maximum degree $\Delta(G)$. The maximum average degree of a graph G, denoted by $mad(G)$, is the maximum of the average degrees of all subgraphs of G. In this note, we shall prove the following. Let G be a graph, (1) if $mad(G) < \frac{8}{3}$ and $\Delta(G) \geq 7$, then $lc_l(G) = \left\lceil \frac{\Delta(G)}{2} \right\rceil + 1$; (2) if $mad(G) < \frac{18}{7}$ and $\Delta(G) \geq 5$, then $lc_l(G) = \left\lceil \frac{\Delta(G)}{2} \right\rceil + 1$; (3) if $mad(G) < \frac{20}{7}$ and $\Delta(G) \geq 5$, then $lc_l(G) \leq \left\lceil \frac{\Delta(G)}{2} \right\rceil + 2$.

Keywords: linear coloring, maximum average degree, planar graphs, discharging.

2010 Mathematics Subject Classification: 05C15.

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Received 12 May 2017
Revised 31 January 2018
Accepted 31 July 2018