

GENERALIZED HYPERGRAPH COLORING

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Abstract

A smooth hypergraph property $\mathcal{P}$ is a class of hypergraphs that is hereditary and non-trivial, i.e., closed under induced subhypergraphs and it contains a non-empty hypergraph but not all hypergraphs. In this paper we examine $\mathcal{P}$ -colorings of hypergraphs with smooth hypergraph properties $\mathcal{P}$. A $\mathcal{P}$ -coloring of a hypergraph H with color set C is a function $\varphi : V(H) \rightarrow C$ such that $H[\varphi^{-1}(c)]$ belongs to $\mathcal{P}$ for all $c \in C$. Let $L : V(H) \rightarrow 2^C$ be a so called list-assignment of the hypergraph H . Then, a $(\mathcal{P}, L)$ -coloring of H is a $\mathcal{P}$ -coloring φ of H such that $\varphi(v) \in L(v)$ for all $v \in V(H)$. The aim of this paper is a characterization of $(\mathcal{P}, L)$ -critical hypergraphs. Those are hypergraphs H such that $H - v$ is $(\mathcal{P}, L)$ -colorable for all $v \in V(H)$ but H itself is not. Our main theorem is a Gallai-type result for critical hypergraphs, which implies a Brooks-type result for $(\mathcal{P}, L)$ -colorable hypergraphs. In the last section, we prove a Gallai-type bound for the degree sum of $(\mathcal{P}, L)$ -critical locally simple hypergraphs.

Keywords: hypergraph decomposition, vertex partition, degeneracy, coloring of hypergraphs, hypergraph properties.

2010 Mathematics Subject Classification: 05C15.

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¹Partially supported by DAAD, Germany (as part of BMBF) and by the Ministry of Education Science, Research and Sport of the Slovak Republic within the project 57320575.

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Received 18 April 2018
Revised 23 August 2018
Accepted 23 August 2018