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GENERALIZED HYPERGRAPH COLORING

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Abstract

A smooth hypergraph property $\mathcal{P}$ is a class of hypergraphs that is hereditary and non-trivial, i.e., closed under induced subhypergraphs and it contains a non-empty hypergraph but not all hypergraphs. In this paper we examine $\mathcal{P}$-colorings of hypergraphs with smooth hypergraph properties $\mathcal{P}$. A $\mathcal{P}$-coloring of a hypergraph H with color set C is a function $\varphi : V(H) \to C$ such that $H[\varphi^{-1}(c)]$ belongs to $\mathcal{P}$ for all $c \in C$. Let $L : V(H) \to 2^C$ be a so called list-assignment of the hypergraph H. Then, a $(\mathcal{P}, L)$-coloring of H is a $\mathcal{P}$-coloring φ of H such that $\varphi(v) \in L(v)$ for all $v \in V(H)$. The aim of this paper is a characterization of $(\mathcal{P}, L)$-critical hypergraphs. Those are hypergraphs H such that $H - v$ is $(\mathcal{P}, L)$-colorable for all $v \in V(H)$ but H itself is not. Our main theorem is a Gallai-type result for critical hypergraphs, which implies a Brooks-type result for $(\mathcal{P}, L)$-colorable hypergraphs. In the last section, we prove a Gallai-type bound for the degree sum of $(\mathcal{P}, L)$-critical locally simple hypergraphs.

Keywords: hypergraph decomposition, vertex partition, degeneracy, coloring of hypergraphs, hypergraph properties.

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References

[1] M. Borowiecki, I. Broere, M. Frick, P. Mihók and G. Semanišin, *A survery of hereditary properties of graphs*, Discuss. Math. Graph Theory **17** (1997) 5–50. doi:10.7151/dmgt.1037

[2] M. Borowiecki, I. Broere and P. Mihók, *On generalized list colourings of graphs*, Discuss. Math. Graph Theory **17** (1995) 127–132. doi:10.7151/dmgt.1045

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[3] M. Borowiecki, E. Drgas-Burchardt and P. Mihók, *Generalized list colouring of graphs*, Discuss. Math. Graph Theory **15** (1995) 185–193.
doi:10.7151/dmgt.1016

[4] R.L. Brooks, *On colouring the nodes of a network*, Proc. Cambridge Philos. Soc., Math. Phys. Sci. **37** (1941) 194–197.

[5] E.J. Cockayne, *Colour classes for r-graphs*, Canad. Math. Bull. **15** (1972) 349–354.
doi:10.4153/CMB-1972-063-2

[6] G.A. Dirac, *A property of* 4*-chromatic graphs and some remarks on critical graphs*, J. Lond. Math. Soc. **27** (1952) 85–92.
doi:10.1112/jlms/s1-27.1.85

[7] G.A. Dirac, *The structure of k-chromatic graphs*, Fund. Math. **40** (1953) 42–55.
doi:10.4064/fm-40-1-42-55

[8] P. Erdős and A. Hajnal, *On the chromatic number of graphs and set-systems*, Acta Math. Acad. Sci. Hungar. **17** (1966) 61–99.
doi:10.1007/BF02020444

[9] P. Erdős, A.L. Rubin and H. Taylor, *Choosability in graphs*, Congr. Numer. **XXVI** (1979) 125–157.

[10] T. Gallai, *Kritische Graphen* II, Publ. Math. Inst. Hungar. Acad. Sci. **8** (1963) 373–395.

[11] R.P. Jones, *Brooks' theorem for hypergraphs*, in: Proc. 5th British Combinatorial Conf. (1975) 379–384.

[12] A.V. Kostochka and M. Stiebitz, *A new lower bound on the number of edges in colour-critical graphs and hypergraphs*, J. Combin. Theory Ser. B **87** (2003) 374–402.
doi:10.1016/S0095-8956(02)00035-7

[13] A.V. Kostochka, M. Stiebitz and B. Wirth, *The colour theorems of Brooks and Gallai extended*, Discrete Math. **191** (1996) 125–137.
doi:10.1016/S0012-365X(98)00100-9

[14] L. Lovász, *On chromatic number of finite set-systems*, Acta Math. Acad. Sci. Hungar. **19** (1968) 59–67.
doi:10.1007/BF01894680

[15] P. Mihók and R. Škrekovski, *Gallai's inequality for critical graphs of reducible hereditary properties*, Discuss. Math. Graph Theory **21** (2001) 167–177.
doi:10.7151/dmgt.1141

[16] T. Schweser and M. Stiebitz, *Hypergraph partitions and variable degeneracy*, (2017) preprint.
arXiv:1804.04894

[17] V.G. Vizing, *Vertex coloring with given colors*, Diskret. Analiz. **29** (1976) 3–10, in Russian.

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