

ON THE n -PARTITE TOURNAMENTS WITH EXACTLY $n - m + 1$ CYCLES OF LENGTH m

QIAOPING GUO AND WEI MENG

*School of Mathematical Sciences
Shanxi University, Taiyuan, 030006, China*

e-mail: guoqp@sxu.edu.cn
mengwei@sxu.edu.cn

Abstract

Gutin and Rafiey [*Multipartite tournaments with small number of cycles*, Australas J. Combin. 34 (2006) 17–21] raised the following two problems: (1) Let $m \in \{3, 4, \dots, n\}$. Find a characterization of strong n -partite tournaments having exactly $n - m + 1$ cycles of length m ; (2) Let $3 \leq m \leq n$ and $n \geq 4$. Are there strong n -partite tournaments, which are not themselves tournaments, with exactly $n - m + 1$ cycles of length m for two values of m ? In this paper, we discuss the strong n -partite tournaments D containing exactly $n - m + 1$ cycles of length m for $4 \leq m \leq n - 1$. We describe the substructure of such D satisfying a given condition and we also show that, under this condition, the second problem has a negative answer.

Keywords: multipartite tournaments, tournaments, cycles.

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