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# CONGRUENCES AND HOEHNKE RADICALS ON GRAPHS

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**Abstract**

We motivate, introduce, and study radicals on classes of graphs. This concept, and the theory which is developed, imitates the original notion of a Hoehnke radical in universal algebra using congruences. It is shown how this approach ties in with the existing theory of connectednesses and disconnectednesses (= Kurosh-Amitsur radical theory).

**Keywords:** congruences and quotients of graphs, Hoehnke radicals of graphs, connectednesses and disconnectednesses of graphs, Kurosh-Amitsur radicals of graphs, subdirect representations of graphs.

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## References

[1] G. Birkhoff, *Subdirect unions in universal algebra*, Bull. Amer. Math. Soc. **50** (1944) 764–768.
doi:10.1090/S0002-9904-1944-08235-9

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[2] I. Broere and J. Heidema, *Products and tensor products of graphs and homomorphisms.*
arXiv:1612.00601 [math.CO]

[3] I. Broere, J. Heidema and M.D.V. Matsoha, *The quest for a characterization of hom-properties of finite character*, Discuss. Math. Graph Theory **36** (2016) 479–500.
doi:10.7151/dmgt.1873

[4] I. Broere, J. Heidema and L.M. Pretorius, *Graph congruences and what they connote*, Quaest. Math., to appear.
doi:10.2989/16073606.2017.1419388

[5] A. Buys and J. Heidema, *Model-theoretical aspects of radicals and subdirect decomposability*, in: M.J. Schoeman Ed, Proceedings of the Second Algebra Symposium held at the University of Pretoria in August 1980, Department of Mathematics, University of Pretoria, Pretoria **484** (1982) 21–25.

[6] A. Buys and J. Heidema, *Radicals and subdirect products in models*, Comm. Algebra **10** (1982) 1315–1330.
doi:10.1080/00927878208822778

[7] B.A. Davey and H.A. Priestly, Introduction to Lattices and Order, Second Edition (Cambridge University Press, New York, 2008).

[8] R. Diestel, Graph Theory (Springer, Heidelberg, 2010).
doi:10.1007/978-3-642-14279-6

[9] E. Fried and R. Wiegandt, *Connectednesses and disconnectednesses of graphs*, Algebra Universalis **5** (1975) 411–428.
doi:10.1007/BF02485276

[10] B.J. Gardner and R. Wiegandt, Radical Theory of Rings (CRC Press, 2003).

[11] M. Gray, A Radical Approach to Algebra (Addison-Wesley Publishing Co., Mass.-London-Don Mills, Ontario, 1970).

[12] H.-J. Hoehnke, *Einige neue Resultate über abstrakte Halbgruppen*, Colloq. Math. **14** (1966) 329–348.
doi:10.4064/cm-14-1-329-348

[13] H.-J. Hoehnke, *Radikale in allgemeinen Algebren*, Math. Nachr. **32** (1966) 347–383.
doi:10.1002/mana.19660320607

[14] G. Köthe, *Die Struktur der Ringe, deren Restklassenring nach dem Radikal vollständig reduzibel ist*, Math. Z. **32** (1930) 161–186.
doi:10.1007/BF01194626

[15] L. Márki, R. Mlitz and R.A. Wiegandt, *A general Kurosh-Amitsur radical theory*, Comm. Algebra **16** (1988) 249–305.
doi:10.1080/00927878808823572

[16] A. Robinson, Introduction to Model Theory and to the Metamathematics of Algebra (North-Holland, Amsterdam, 1963).

[17] S. Veldsman, A General Radical Theory in Categories, Ph.D. Thesis (University of Port Elizabeth, 1980).

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