

CONGRUENCES AND HOEHNKE RADICALS ON GRAPHS

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Abstract

We motivate, introduce, and study radicals on classes of graphs. This concept, and the theory which is developed, imitates the original notion of a Hoehnke radical in universal algebra using congruences. It is shown how this approach ties in with the existing theory of connectednesses and disconnectednesses (= Kurosh-Amitsur radical theory).

Keywords: congruences and quotients of graphs, Hoehnke radicals of graphs, connectednesses and disconnectednesses of graphs, Kurosh-Amitsur radicals of graphs, subdirect representations of graphs.

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