

DOMINATION NUMBER, INDEPENDENT DOMINATION NUMBER AND 2-INDEPENDENCE NUMBER IN TREES

NASRIN DEHGARDI¹

Department of Mathematics and Computer Science
Sirjan University of Technology, Sirjan, I.R. Iran

e-mail: n.dehgardi@sirjantech.ac.ir

SEYED MAHMOUD SHEIKHOLESAMI, MINA VALINAVAZ

Department of Mathematics
Azarbaijan Shahid Madani University, Tabriz, I.R. Iran

e-mail: s.m.sheikholesami
m.valinavaz@azaruniv.ac.ir

HAMIDEH ARAM

Department of Mathematics
Gareziaeddin Center, Khoy Branch
Islamic Azad University, Khoy, Iran

e-mail: hamideh.aram@gmail.com

AND

LUTZ VOLKMANN

Lehrstuhl II für Mathematik
RWTH Aachen University, 52056 Aachen, Germany

e-mail: volkm@math2.rwth-aachen.de

Abstract

For a graph G , let $\gamma(G)$ be the domination number, $i(G)$ be the independent domination number and $\beta_2(G)$ be the 2-independence number. In this paper, we prove that for any tree T of order $n \geq 2$, $4\beta_2(T) - 3\gamma(T) \geq 3i(T)$, and we characterize all trees attaining equality. Also we prove that for every tree T of order $n \geq 2$, $i(T) \leq \frac{3\beta_2(T)}{4}$, and we characterize all extreme trees.

¹Corresponding author.

Keywords: 2-independence number, domination number, independent domination number.

2010 Mathematics Subject Classification: 05C69.

REFERENCES

- [1] J. Amjadi, N. Dehgardi, S.M. Sheikholeslami and M. Valinavaz, *Independent Roman domination and 2-independence in trees*, Discrete Math. Algorithms Appl. **10** (2018) 1850052.
doi:10.1142/S1793830918500520
- [2] M. Chellali, O. Favaron, A. Hansberg and L. Volkmann, *k-domination and k-independence in graphs: A survey*, Graphs Combin. **28** (2012) 1–55.
doi:10.1007/s00373-011-1040-3
- [3] M. Chellali and N. Meddah, *Trees with equal 2-domination and 2-independence numbers*, Discuss. Math. Graph Theory **32** (2012) 263–270.
doi:10.7151/dmgt.1603
- [4] N. Dehgardi, *Mixed Roman domination and 2-independence in trees*, Commun. Comb. Optim. **3** (2018) 79–91.
doi:10.22049/CCO.2018.25964.1062
- [5] O. Favaron, *On a conjecture of Fink and Jacobson concerning k-domination and k-dependence*, J. Combin. Theory Ser. B **39** (1985) 101–102.
doi:10.1016/0095-8956(85)90040-1
- [6] O. Favaron, *Graduate course in the University of Blida* (2005), unpublished.
- [7] J.F. Fink and M.S. Jacobson, *n-domination in graphs*, in: Graph Theory with Applications to Algorithms and Computer Science (Wiley, New York, 1985) 283–300.
- [8] Fink and M.S. Jacobson, *On n-domination, n-dependence and forbidden subgraphs*, in: Graph Theory with Applications to Algorithms and Computer Science (Wiley, New York, 1985) 301–311.
- [9] A. Hansberg, D. Meierling and L. Volkmann, *Independence and k-domination in graphs*, Int. J. Comput. Math. **88** (2011) 905–915.
doi:10.1080/00207160.2010.482664
- [10] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, *Fundamentals of Domination in Graphs* (Marcel Dekker, Inc., New York, 1998).
- [11] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, *Domination in Graphs: Advanced Topics* (Marcel Dekker, Inc., New York, 1998).
- [12] M.S. Jacobson, K. Peters and D.F. Rall, *On n-irredundance and n-domination*, Ars Combin. **29** (1990) 151–160.
- [13] N. Meddah and M. Chellali, *Roman domination and 2-independence in trees*, Discrete Math. Algorithms Appl. **9** (2017) 1750023.
doi:10.1142/S1793830917500239

Received 4 July 2017
Revised 7 July 2018
Accepted 9 July 2018