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DOMINATION NUMBER, INDEPENDENT DOMINATION NUMBER AND 2-INDEPENDENCE NUMBER IN TREES

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Abstract

For a graph G, let $\gamma(G)$ be the domination number, $i(G)$ be the independent domination number and $\beta_2(G)$ be the 2-independence number. In this paper, we prove that for any tree T of order $n \geq 2$, $4\beta_2(T) - 3\gamma(T) \geq 3i(T)$, and we characterize all trees attaining equality. Also we prove that for every tree T of order $n \geq 2$, $i(T) \leq \frac{3\beta_2(T)}{4}$, and we characterize all extreme trees.

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