

TREES WITH DISTINGUISHING INDEX EQUAL DISTINGUISHING NUMBER PLUS ONE

SAEID ALIKHANI

Department of Mathematics, Yazd University, 89195-741, Yazd, Iran

e-mail: alikhani@yazd.ac.ir

SANDI KLAVŽAR

Faculty of Mathematics and Physics, University of Ljubljana, Slovenia
Faculty of Natural Sciences and Mathematics, University of Maribor, Slovenia
Institute of Mathematics, Physics and Mechanics, Ljubljana, Slovenia

e-mail: sandi.klavzar@fmf.uni-lj.si

FLORIAN LEHNER

Mathematics Institute, University of Warwick, Coventry, United Kingdom

e-mail: mail@florian-lehner.net

AND

SAMANEH SOLTANI

Department of Mathematics, Yazd University, 89195-741, Yazd, Iran

e-mail: s.soltani1979@gmail.com

Abstract

The distinguishing number (index) $D(G)$ ($D'(G)$) of a graph G is the least integer d such that G has an vertex (edge) labeling with d labels that is preserved only by the trivial automorphism. It is known that for every graph G we have $D'(G) \leq D(G) + 1$. In this note we characterize finite trees for which this inequality is sharp. We also show that if G is a connected unicyclic graph, then $D'(G) = D(G)$.

Keywords: automorphism group, distinguishing index, distinguishing number, tree, unicyclic graph.

2010 Mathematics Subject Classification: 05C15, 05E18.

REFERENCES

- [1] M.O. Albertson and K.L. Collins, *Symmetry breaking in graphs*, Electron. J. Combin. **3** (1996) #R18.
- [2] S. Alikhani and S. Soltani, *Distinguishing number and distinguishing index of certain graphs*, Filomat **31** (2017) 4393–4404.
doi:10.2298/FIL1714393A
- [3] V. Arvind, C. Cheng and N. Devanur, *On computing the distinguishing numbers of planar graphs and beyond: a counting approach*, SIAM J. Discrete Math. **22** (2008) 1297–1324.
doi:10.1137/07068686X
- [4] V. Arvind and N. Devanur, *Symmetry breaking in trees and planar graphs by vertex coloring*, in: Proceedings of the 8th Nordic Combinatorial Conference (Aalborg University, Aalborg, Denmark, 2004).
- [5] M. Cavers and K. Seyffarth, *Graphs with large distinguishing chromatic number*, Electron. J. Combin. **20** (2013) #P19.
- [6] M. Chan, *The distinguishing number of the augmented cube and hypercube powers*, Discrete Math. **308** (2008) 2330–2336.
doi:10.1016/j.disc.2006.09.056
- [7] C. Cheng, *On computing the distinguishing numbers of trees and forests*, Electron. J. Combin. **13** (2006) #R11.
- [8] K.L. Collins and A.N. Trenk, *The distinguishing chromatic number*, Electron. J. Combin. **13** (2016) #R16.
- [9] E. Estaji, W. Imrich, R. Kalinowski, M. Pilśniak and T. Tucker, *Distinguishing Cartesian products of countable graphs*, Discuss. Math. Graph Theory **37** (2017) 155–164.
doi:10.7151/dmgt.1902
- [10] S. Gravier, K. Meslem, S. Schmidt and S. Slimani, *A new game invariant of graphs: the game distinguishing number*, Discrete Math. Theor. Comput. Sci. **19** (1) (2017) Paper No. 2.
doi:10.23638/DMTCS-19-1-2
- [11] P. Immel and P.S. Wenger, *The list distinguishing number equals the distinguishing number for interval graphs*, Discuss. Math. Graph Theory **37** (2017) 165–174.
doi:10.7151/dmgt.1927
- [12] W. Imrich, R. Kalinowski, M. Pilśniak and M.H. Shekarriz, *Bounds for distinguishing invariants of infinite graphs*, Electron. J. Combin. **24** (2017) #P3.6.
- [13] R. Kalinowski and M. Pilśniak, *Distinguishing graphs by edge colourings*, European J. Combin. **45** (2015) 124–131.
doi:10.1016/j.ejc.2014.11.003
- [14] S. Klavžar, T.-L. Wong and X. Zhu, *Distinguishing labelings of group action on vector spaces and graphs*, J. Algebra **303** (2006) 626–641.
doi:10.1016/j.jalgebra.2006.01.045

- [15] D. Kim, Y.S. Kwon and J. Lee, *The distinguishing numbers of Merged Johnson graphs*, Bull. Korean Math. Soc. **52** (2015) 395–408.
doi:10.4134/BKMS.2015.52.2.395
- [16] C. Laflamme, L. Nguyen Van Thé and N. Sauer, *Distinguishing number of countable homogeneous relational structures*, Electron. J. Combin. **17** (2010) #R20.
- [17] F. Lehner, *Distinguishing graphs with intermediate growth*, Combinatorica **36** (2016) 333–347.
doi:10.1007/s00493-015-3071-5
- [18] F. Lehner, *Breaking graph symmetries by edge colourings*, J. Combin. Theory Ser. B **127** (2017) 205–214.
doi:10.1016/j.jctb.2017.06.001
- [19] F. Lehner and S.M. Smith, *On symmetries of edge and vertex colourings of graphs*, preprint.
- [20] R. Schmidt, *Ein Ordnungsbegriff für Graphen ohne unendliche Wege mit einer Anwendung auf n -fach zusammenhängende Graphen*, Arch. Math. **40** (1983) 283–288.
doi:10.1007/BF01192782
- [21] S.M. Smith and M.E. Watkins, *Bounding the distinguishing number of infinite graphs and permutation groups*, Electron. J. Combin. **21** (2014) #P3.40.
- [22] M. Watkins and X. Zhou, *Distinguishability of locally finite trees*, Electron. J. Combin. **14** (2007) #R29.
- [23] T.-L. Wong and X. Zhu, *Distinguishing labeling of group actions*, Discrete Math. **309** (2009) 1760–1765.
doi:10.1016/j.disc.2008.02.022

Received 15 September 2017

Revised 4 July 2018

Accepted 13 July 2018