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THE DOUBLE ROMAN DOMATIC NUMBER OF A DIGRAPH

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Abstract

A *double Roman dominating function* on a digraph D with vertex set $V(D)$ is defined in [G. Hao, X. Chen and L. Volkmann, *Double Roman domination in digraphs*, Bull. Malays. Math. Sci. Soc. (2017).] as a function $f : V(D) \to \{0, 1, 2, 3\}$ having the property that if $f(v) = 0$, then the vertex v must have at least two in-neighbors assigned 2 under f or one in-neighbor w with $f(w) = 3$, and if $f(v) = 1$, then the vertex v must have at least one in-neighbor u with $f(u) \geq 2$. A set $\{f_1, f_2, \ldots, f_d\}$ of distinct double Roman dominating functions on D with the property that $\sum_{i=1}^{d} f_i(v) \leq 3$ for each $v \in V(D)$ is called a *double Roman dominating family* (of functions) on D. The maximum number of functions in a double Roman dominating family on D is the *double Roman domatic number* of D, denoted by $d_{dR}(D)$. We initiate the study of the double Roman domatic number, and we present different sharp bounds on $d_{dR}(D)$. In addition, we determine the double Roman domatic number of some classes of digraphs.

Keywords: digraph, double Roman domination, double Roman domatic number.

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