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THE LIST EDGE COLORING AND LIST TOTAL COLORING OF PLANAR GRAPHS WITH MAXIMUM DEGREE AT LEAST 7

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Abstract

A graph G is edge k-choosable (respectively, total k-choosable) if, whenever we are given a list $L(x)$ of colors with $|L(x)| = k$ for each $x \in E(G)$ ($x \in E(G) \cup V(G)$), we can choose a color from $L(x)$ for each element x such that no two adjacent (or incident) elements receive the same color. The list edge chromatic index $\chi'_l(G)$ (respectively, the list total chromatic

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number $\chi''_l(G)$) of G is the smallest integer k such that G is edge (respectively, total) k-choosable. In this paper, we focus on a planar graph G, with maximum degree $\Delta(G) \geq 7$ and with some structural restrictions, satisfies $\chi'_l(G) = \Delta(G)$ and $\chi''_l(G) = \Delta(G) + 1$.

Keywords: planar graph, list edge coloring, list total coloring.

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Appendix

```
%Input
syms x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 x12
%Lemma ?? (1)
Q1=(x1-x2)*(x1-x3)*(x1-x4)*(x1-x6)*(x2-x3)*(x2-x4)*(x2-x5)*(x3-x5)*(x3-x6)*(x4-x5)
    *(x4-x6)*(x5-x6);
c1=diff(diff(diff(diff(diff(diff(Q1,x1,3),x2,3),x3,1),x4,1),x5,2),x6,2) /factorial(3)/factorial(3)
    /factorial(1)/factorial(1)/factorial(2)/factorial(2)
%Lemma ?? (2)
Q2=(x1-x2)*(x1-x5)*(x1-x6)*(x1-x7)*(x2-x5)*(x2-x7)*(x2-x8)*(x3-x5)*(x3-x4)*(x3-x9)
    *(x3-x10)*(x4-x5)*(x4-x10)*(x4-x11)*(x5-x6)*(x5-x8)*(x5-x9)*(x5-x11)*(x6-x7)*(x6-x8)
    *(x6-x9)*(x6-x11)*(x7-x8)*(x8-x9)*(x8-x11)*(x9-x10)*(x9-x11)*(x10-x11);
c2=diff(diff(diff(diff(diff(diff(diff(diff(diff(diff(diff(Q1,x1,3),x2,0),x3,0), x4,3),x5,5),x6,4),x7,2),
    x8,2),x9,3),x10,2),x11,4)/factorial(3)/factorial(0)/factorial(0) /factorial(3)/factorial(5)
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/factorial(4)/factorial(2)/factorial(2)/factorial(3)/factorial(2)/factorial(4)

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%Lemma ?? (3)
Q3=(x1-x2)*(x1-x3)*(x1-x4)*(x1-x5)*(x1-x9)*(x1-x8)*(x2-x3)*(x2-x5)*(x2-x6)*(x3-x4)
    *(x3-x6)*(x3-x9)*(x3-x7)*(x4-x7)*(x4-x8)*(x5-x9)*(x5-x6)*(x5-x8)*(x6-x7)*(x6-x9)
    *(x7-x8)*(x7-x9)*(x8-x9);
c3=diff(diff(diff(diff(diff(diff(diff(diff(diff(Q1,x1,3),x2,3),x3,3),x4,3),x5,3), x6,3),x7,3),
    x8,2),x9,0)/factorial(3)/factorial(3)/factorial(3)/factorial(3)/factorial(3)
    /factorial(3)/factorial(3)/factorial(2)
```