Discussiones Mathematicae
Graph Theory 40 (2020) 1025–1033
doi:10.7151/dmgt.2159

LOW 5-STARS AT 5-VERTICES IN 3-POLYTOPES WITH MINIMUM DEGREE 5 AND NO VERTICES OF DEGREE FROM 7 TO 9 [1]

Oleg V. Borodin, Mikhail A. Bykov

and

Anna O. Ivanova

Sobolev Institute of Mathematics
Novosibirsk, 630090, Russia

e-mail: brdnoleg@math.nsc.ru
131093@mail.ru
shmgnanna@mail.ru

Abstract

In 1940, Lebesgue gave an approximate description of the neighborhoods of 5-vertices in the class $\mathbf{P_5}$ of 3-polytopes with minimum degree 5.

Given a 3-polytope P, by $h_5(P)$ we denote the minimum of the maximum degrees (height) of the neighborhoods of 5-vertices (minor 5-stars) in P.

Recently, Borodin, Ivanova and Jensen showed that if a polytope P in $\mathbf{P_5}$ is allowed to have a 5-vertex adjacent to two 5-vertices and two more vertices of degree at most 6, called a $(5,5,6,6,\infty)$-vertex, then $h_5(P)$ can be arbitrarily large. Therefore, we consider the subclass $\mathbf{P_5^*}$ of 3-polytopes in $\mathbf{P_5}$ that avoid $(5,5,6,6,\infty)$-vertices.

For each P^* in $\mathbf{P_5^*}$ without vertices of degree from 7 to 9, it follows from Lebesgue's Theorem that $h_5(P^*) \le 17$. Recently, this bound was lowered by Borodin, Ivanova, and Kazak to the sharp bound $h_5(P^*) \le 15$ assuming the absence of vertices of degree from 7 to 11 in P^*.

In this note, we extend the bound $h_5(P^*) \le 15$ to all P^*s without vertices of degree from 7 to 9.

Keywords: planar map, planar graph, 3-polytope, structural properties, 5-star, weight, height.

2010 Mathematics Subject Classification: 05C75.

[1]The work was funded by the Russian Science Foundation, grant 16-11-10054.

References

[1] O.V. Borodin, *Structural properties of planar maps with the minimal degree* 5, Math. Nachr. **158** (1992) 109–117. doi:10.1002/mana.19921580108

[2] O.V. Borodin and A.O. Ivanova, *Describing* 4*-stars at* 5*-vertices in normal plane maps with minimum degree* 5, Discrete Math. **313** (2013) 1710–1714. doi:10.1016/j.disc.2013.04.025

[3] O.V. Borodin and A.O. Ivanova, *Light neighborhoods of* 5*-vertices in* 3*-polytopes with minimum degree* 5, Sib. Èlektron. Mat. Izv. **13** (2016) 584–591. doi:10.17377/semi.2016.13.045

[4] O.V. Borodin and A.O. Ivanova, *Light and low* 5*-stars in normal plane maps with minimum degree* 5, Sib. Math. J. **57** (2016) 470–475. doi:10.1134/S0037446616030071

[5] O.V. Borodin, A.O. Ivanova and T.R. Jensen, 5*-stars of low weight in normal plane maps with minimum degree* 5, Discuss. Math. Graph Theory **34** (2014) 539–546. doi:10.7151/dmgt.1748

[6] O.V. Borodin, A.O. Ivanova and O.N. Kazak, *Describing neighborhoods of* 5*-vertices in* 3*-polytopes with minimum degree* 5 *and without vertices of degrees from* 7 *to* 11, Discuss. Math. Graph Theory **38** (2018) 615–625. doi:10.7151/dmgt.2024

[7] O.V. Borodin, A.O. Ivanova, O.N. Kazak and E.I. Vasil'eva, *Heights of minor* 5*-stars in* 3*-polytopes with minimum degree* 5 *and no vertices of degree* 6 *and* 7, Discrete Math. **341** (2018) 825–829. doi:10.1016/j.disc.2017.11.021

[8] O.V. Borodin, A.O. Ivanova and D.V. Nikiforov, *Low minor* 5*-stars in* 3*-polytopes with minimum degree* 5 *and no* 6*-vertices*, Discrete Math. **340** (2017) 1612–1616. doi:10.1016/j.disc.2017.03.002

[9] O.V. Borodin, A.O. Ivanova and D.V. Nikiforov, *Low and light* 5*-stars in* 3*-polytopes with minimum degree* 5 *and restrictions on the degrees of major vertices*, Sib. Math. J. **58** (2017) 600–605. doi:10.1134/S003744661704005X

[10] O.V. Borodin and D.R. Woodall, *Short cycles of low weight in normal plane maps with minimum degree* 5, Discuss. Math. Graph Theory **18** (1998) 159–164. doi:10.7151/dmgt.1071

[11] P. Franklin, *The four color problem*, Amer. J. Math. **44** (1922) 225–236. doi:10.2307/2370527

[12] S. Jendrol' and T. Madaras, *On light subgraphs in plane graphs of minimum degree five*, Discuss. Math. Graph Theory **16** (1996) 207–217. doi:10.7151/dmgt.1035

[13] H. Lebesgue, *Quelques conséquences simples de la formule d'Euler*, J. Math. Pures Appl. **19** (9) (1940) 27–43.

[14] E. Steinitz, *Polyeder und Raumeinteilungen*, in: Enzykl. Math. Wiss. (Geometrie), **3** (1922) 1–139.

[15] P. Wernicke, *Über den kartographischen Vierfarbensatz*, Math. Ann. **58** (1904) 413–426.
doi:10.1007/BF01444968

Received 18 December 2017
Revised 25 June 2018
Accepted 25 June 2018