

LOW 5-STARS AT 5-VERTICES IN 3-POLYTOPES WITH
MINIMUM DEGREE 5 AND NO VERTICES OF DEGREE
FROM 7 TO 9¹

OLEG V. BORODIN, MIKHAIL A. BYKOV

AND

ANNA O. IVANOVA

Sobolev Institute of Mathematics
Novosibirsk, 630090, Russia

e-mail: brdnoleg@math.nsc.ru
131093@mail.ru
shmgnanna@mail.ru

Abstract

In 1940, Lebesgue gave an approximate description of the neighborhoods of 5-vertices in the class \mathbf{P}_5 of 3-polytopes with minimum degree 5.

Given a 3-polytope P , by $h_5(P)$ we denote the minimum of the maximum degrees (height) of the neighborhoods of 5-vertices (minor 5-stars) in P .

Recently, Borodin, Ivanova and Jensen showed that if a polytope P in \mathbf{P}_5 is allowed to have a 5-vertex adjacent to two 5-vertices and two more vertices of degree at most 6, called a $(5, 5, 6, 6, \infty)$ -vertex, then $h_5(P)$ can be arbitrarily large. Therefore, we consider the subclass \mathbf{P}_5^* of 3-polytopes in \mathbf{P}_5 that avoid $(5, 5, 6, 6, \infty)$ -vertices.

For each P^* in \mathbf{P}_5^* without vertices of degree from 7 to 9, it follows from Lebesgue's Theorem that $h_5(P^*) \leq 17$. Recently, this bound was lowered by Borodin, Ivanova, and Kazak to the sharp bound $h_5(P^*) \leq 15$ assuming the absence of vertices of degree from 7 to 11 in P^* .

In this note, we extend the bound $h_5(P^*) \leq 15$ to all P^* s without vertices of degree from 7 to 9.

Keywords: planar map, planar graph, 3-polytope, structural properties, 5-star, weight, height.

2010 Mathematics Subject Classification: 05C75.

¹The work was funded by the Russian Science Foundation, grant 16-11-10054.

REFERENCES

- [1] O.V. Borodin, *Structural properties of planar maps with the minimal degree 5*, Math. Nachr. **158** (1992) 109–117.
doi:10.1002/mana.19921580108
- [2] O.V. Borodin and A.O. Ivanova, *Describing 4-stars at 5-vertices in normal plane maps with minimum degree 5*, Discrete Math. **313** (2013) 1710–1714.
doi:10.1016/j.disc.2013.04.025
- [3] O.V. Borodin and A.O. Ivanova, *Light neighborhoods of 5-vertices in 3-polytopes with minimum degree 5*, Sib. Elektron. Mat. Izv. **13** (2016) 584–591.
doi:10.17377/semi.2016.13.045
- [4] O.V. Borodin and A.O. Ivanova, *Light and low 5-stars in normal plane maps with minimum degree 5*, Sib. Math. J. **57** (2016) 470–475.
doi:10.1134/S0037446616030071
- [5] O.V. Borodin, A.O. Ivanova and T.R. Jensen, *5-stars of low weight in normal plane maps with minimum degree 5*, Discuss. Math. Graph Theory **34** (2014) 539–546.
doi:10.7151/dmgt.1748
- [6] O.V. Borodin, A.O. Ivanova and O.N. Kazak, *Describing neighborhoods of 5-vertices in 3-polytopes with minimum degree 5 and without vertices of degrees from 7 to 11*, Discuss. Math. Graph Theory **38** (2018) 615–625.
doi:10.7151/dmgt.2024
- [7] O.V. Borodin, A.O. Ivanova, O.N. Kazak and E.I. Vasil'eva, *Heights of minor 5-stars in 3-polytopes with minimum degree 5 and no vertices of degree 6 and 7*, Discrete Math. **341** (2018) 825–829.
doi:10.1016/j.disc.2017.11.021
- [8] O.V. Borodin, A.O. Ivanova and D.V. Nikiforov, *Low minor 5-stars in 3-polytopes with minimum degree 5 and no 6-vertices*, Discrete Math. **340** (2017) 1612–1616.
doi:10.1016/j.disc.2017.03.002
- [9] O.V. Borodin, A.O. Ivanova and D.V. Nikiforov, *Low and light 5-stars in 3-polytopes with minimum degree 5 and restrictions on the degrees of major vertices*, Sib. Math. J. **58** (2017) 600–605.
doi:10.1134/S003744661704005X
- [10] O.V. Borodin and D.R. Woodall, *Short cycles of low weight in normal plane maps with minimum degree 5*, Discuss. Math. Graph Theory **18** (1998) 159–164.
doi:10.7151/dmgt.1071
- [11] P. Franklin, *The four color problem*, Amer. J. Math. **44** (1922) 225–236.
doi:10.2307/2370527
- [12] S. Jendrol' and T. Madaras, *On light subgraphs in plane graphs of minimum degree five*, Discuss. Math. Graph Theory **16** (1996) 207–217.
doi:10.7151/dmgt.1035
- [13] H. Lebesgue, *Quelques conséquences simples de la formule d'Euler*, J. Math. Pures Appl. **19** (9) (1940) 27–43.

- [14] E. Steinitz, *Polyeder und Raumeinteilungen*, in: Enzykl. Math. Wiss. (Geometrie), **3** (1922) 1–139.
- [15] P. Wernicke, *Über den kartographischen Vierfarbensatz*, Math. Ann. **58** (1904) 413–426.
doi:10.1007/BF01444968

Received 18 December 2017

Revised 25 June 2018

Accepted 25 June 2018