

STRONG TUTTE TYPE CONDITIONS AND FACTORS OF GRAPHS

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Abstract

Let $\text{odd}(G)$ denote the number of odd components of a graph G and $k \geq 2$ be an integer. We give sufficient conditions using $\text{odd}(G - S)$ for a graph G to have an even factor. Moreover, we show that if a graph G satisfies $\text{odd}(G - S) \leq \max\{1, (1/k)|S|\}$ for all $S \subset V(G)$, then G has a $(k - 1)$ -regular factor for $k \geq 3$ or an **H**-factor for $k = 2$, where we say that G has an **H**-factor if for every labeling $h : V(G) \rightarrow \{\text{red}, \text{blue}\}$ with $\#\{v \in V(G) : f(v) = \text{red}\}$ even, G has a spanning subgraph F such that $\deg_F(x) = 1$ if $h(x) = \text{red}$ and $\deg_F(x) \in \{0, 2\}$ otherwise.

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REFERENCES

- [1] A. Amahashi, *On factors with all degrees odd*, Graphs Combin. **1** (1985) 111–114.
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- [2] A. Amahashi and M. Kano, *On factors with given components*, Discrete Math. **42** (1982) 1–6.
doi:10.1016/0012-365X(82)90048-6
- [3] J. Akiyama and M. Kano, *Factors and Factorizations of Graphs*, in: Lecture Notes in Math. **2031**, (Springer-Verlag, Berlin 2011).
doi:10.1007/978-3-642-21919-1
- [4] Y. Cui and M. Kano, *Some results on odd factors of graphs*, J. Graph Theory **12** (1988) 327–333.
doi:10.1002/jgt.3190120305
- [5] H. Enomoto, B. Jackson, P. Katerinis and A. Saito, *Toughness and the existence of k -factors*, J. Graph Theory **9** (1985) 87–95.
doi:10.1002/jgt.3190090106
- [6] M. Kano, H. Lu and Q. Yu, *Components factors with large components in graphs*, Appl. Math. Lett. **23** (2010) 385–389.
doi:10.1016/j.aml.2009.11.003
- [7] M. Kano and A. Saito, *Star-factors with large components*, Discrete Math. **312** (2012) 2005–2008.
doi:10.1016/j.disc.2012.03.017
- [8] M. Las Vergnas, *An extension of Tutte’s 1-factor theorem*, Discrete Math. **23** (1978) 241–255.
doi:10.1016/0012-365X(78)90006-7
- [9] H. Lu and M. Kano, *Characterization of 1-tough graphs using factors* (2017).
arXiv:1702.05873
- [10] H. Lu and D. Wang, *On Cui-Kano’s characterization problem on graph factors*, J. Graph Theory **74** (2013) 335–343.
doi:10.1002/jgt.21712
- [11] L. Lovász, *Combinatorial Problems and Exercises* (North-Holland, Amsterdam, 1979).
doi:10.1016/C2009-0-09109-0
- [12] W.T. Tutte, *The factorization of linear graphs*, J. Lond. Math. Soc. (2) **22** (1947) 107–111.
doi:10.1112/jlms/s1-22.2.107
- [13] W.T. Tutte, *The 1-factors of oriented graphs*, Proc. Amer. Math. Soc. **4** (1953) 922–931.
doi:10.2307/2031831
- [14] D. West, *Introduction to Graph Theory* (Prentice Hall, 1996).
- [15] Y. Zhang, G. Yan and M. Kano, *Star-like factors with large components*, J. Oper. Res. Soc. China **3** (2015) 81–88.
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