Discussiones Mathematicae
Graph Theory 40 (2020) 1057–1065
doi:10.7151/dmgt.2158

STRONG TUTTE TYPE CONDITIONS AND FACTORS OF GRAPHS

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Abstract

Let $odd(G)$ denote the number of odd components of a graph G and $k \geq 2$ be an integer. We give sufficient conditions using $odd(G-S)$ for a graph G to have an even factor. Moreover, we show that if a graph G satisfies $odd(G-S) \leq \max\{1,(1/k)|S|\}$ for all $S \subset V(G)$, then G has a $(k-1)$-regular factor for $k \geq 3$ or an **H**-factor for $k=2$, where we say that G has an **H**-factor if for every labeling $h : V(G) \to \{\text{red, blue}\}$ with $\#\{v \in V(G) : f(v) = \text{red}\}$ even, G has a spanning subgraph F such that $\deg_F(x) = 1$ if $h(x) = \text{red}$ and $\deg_F(x) \in \{0,2\}$ otherwise.

Keywords: factor of graph, even factor, regular factor, Tutte type condition.

2010 Mathematics Subject Classification: 05C70.

References

[1] A. Amahashi, *On factors with all degrees odd*, Graphs Combin. **1** (1985) 111–114. doi:10.1007/BF02582935

[1]This work was in part supported by the NSFC (11601041), Open Research Fund Program of Institute of Applied Mathematics Yangtze University (KF1601), The Yangtze Youth Fund (70107021).

[2]This work was supported by JSPS KAKENHI Grant Number 16K05248.

[2] A. Amahashi and M. Kano, *On factors with given components*, Discrete Math. **42** (1982) 1–6.
doi:10.1016/0012-365X(82)90048-6

[3] J. Akiyama and M. Kano, Factors and Factorizations of Graphs, in: Lecture Notes in Math. **2031**, (Springer-Verlag, Berlin 2011).
doi:10.1007/978-3-642-21919-1

[4] Y. Cui and M. Kano, *Some results on odd factors of graphs*, J. Graph Theory **12** (1988) 327–333.
doi:10.1002/jgt.3190120305

[5] H. Enomoto, B. Jackson, P. Katerinis and A. Saito, *Toughness and the existence of k-factors*, J. Graph Theory **9** (1985) 87–95.
doi:10.1002/jgt.3190090106

[6] M. Kano, H. Lu and Q. Yu, *Components factors with large components in graphs*, Appl. Math. Lett. **23** (2010) 385–389.
doi:10.1016/j.aml.2009.11.003

[7] M. Kano and A. Saito, *Star-factors with large components*, Discrete Math. **312** (2012) 2005–2008.
doi:10.1016/j.disc.2012.03.017

[8] M. Las Vergnas, *An extension of Tutte's* 1*-factor theorem*, Discrete Math. **23** (1978) 241–255.
doi:10.1016/0012-365X(78)90006-7

[9] H. Lu and M. Kano, *Characterization of* 1*-tough graphs using factors* (2017).
arXiv:1702.05873

[10] H. Lu and D. Wang, *On Cui-Kano's characterization problem on graph factors*, J. Graph Theory **74** (2013) 335–343.
doi:10.1002/jgt.21712

[11] L. Lovász, Combinatorial Problems and Exercises (North-Holland, Amsterdam, 1979).
doi:10.1016/C2009-0-09109-0

[12] W.T. Tutte, *The factorization of linear graphs*, J. Lond. Math. Soc. (2) **22** (1947) 107–111.
doi:10.1112/jlms/s1-22.2.107

[13] W.T. Tutte, *The* 1*-factors of oriented graphs*, Proc. Amer. Math. Soc. **4** (1953) 922–931.
doi:10.2307/2031831

[14] D. West, Introduction to Graph Theory (Prentice Hall, 1996).

[15] Y. Zhang, G. Yan and M. Kano, *Star-like factors with large components*, J. Oper. Res. Soc. China **3** (2015) 81–88.
doi:10.1007/s40305-014-0066-7

Received 26 October 2017
Revised 26 June 2018
Accepted 26 June 2018