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ADDITIVE LIST COLORING OF PLANAR GRAPHS WITH GIVEN GIRTH

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Abstract

An *additive coloring* of a graph G is a labeling of the vertices of G from $\{1, 2, \ldots, k\}$ such that two adjacent vertices have distinct sums of labels on their neighbors. The least integer k for which a graph G has an additive coloring is called the *additive coloring number* of G, denoted $\chi_\Sigma(G)$. Additive coloring is also studied under the names lucky labeling and open distinguishing. In this paper, we improve the current bounds on the additive coloring number for particular classes of graphs by proving results for a list version of additive coloring. We apply the discharging method and the Combinatorial Nullstellensatz to show that every planar graph G with girth at least 5 has $\chi_\Sigma(G) \leq 19$, and for girth at least 6, 7, and 26, $\chi_\Sigma(G)$ is at most 9, 8, and 3, respectively. In 2009, Czerwiński, Grytczuk, and Żelazny conjectured that $\chi_\Sigma(G) \leq \chi(G)$, where $\chi(G)$ is the chromatic number of G. Our result for

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the class of non-bipartite planar graphs of girth at least 26 is best possible and affirms the conjecture for this class of graphs.

Keywords: lucky labeling, additive coloring, reducible configuration, discharging method, Combinatorial Nullstellensatz.

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