Discussiones Mathematicae
Graph Theory 40 (2020) 985–994
doi:10.7151/dmgt.2155

LIGHT MINOR 5-STARS IN 3-POLYTOPES WITH MINIMUM DEGREE 5 AND NO 6-VERTICES [1]

Oleg V. Borodin, Anna O. Ivanova

and

Ekaterina I. Vasil'eva

Institute of Mathematics Siberian Branch
Russian Academy of Sciences
Novosibirsk, 630090, Russia

e-mail: brdnoleg@math.nsc.ru
shmgnanna@mail.ru
ekaterinavasilyeva93@gmail.com

Abstract

In 1940, Lebesgue gave an approximate description of the neighborhoods of 5-vertices in the class $\mathbf{P_5}$ of 3-polytopes with minimum degree 5.

Given a 3-polytope P, by $w(P)$ denote the minimum of the degree-sum (weight) of the neighborhoods of 5-vertices (minor 5-stars) in P.

In 1996, Jendrol' and Madaras showed that if a polytope P in $\mathbf{P_5}$ is allowed to have a 5-vertex adjacent to four 5-vertices, then $w(P)$ can be arbitrarily large.

For each P in $\mathbf{P_5}$ without vertices of degree 6 and 5-vertices adjacent to four 5-vertices, it follows from Lebesgue's Theorem that $w(P) \leq 68$. Recently, this bound was lowered to $w(P) \leq 55$ by Borodin, Ivanova, and Jensen and then to $w(P) \leq 51$ by Borodin and Ivanova.

In this note, we prove that every such polytope P satisfies $w(P) \leq 44$, which bound is sharp.

Keywords: planar map, planar graph, 3-polytope, structural properties, 5-star, weight, height.

2010 Mathematics Subject Classification: 05C15.

References

[1] The work was funded by the Russian Science Foundation, grant 16-11-10054.

[1] O.V. Borodin and A.O. Ivanova, *Describing* 4*-stars at* 5*-vertices in normal plane maps with minimum degree* 5, Discrete Math. **313** (2013) 1710–1714. doi:10.1016/j.disc.2013.04.025

[2] O.V. Borodin and A.O. Ivanova, *Light and low* 5*-stars in normal plane maps with minimum degree* 5, Sib. Math. J. **57** (2016) 470–475. doi:10.1134/S0037446616030071

[3] O.V. Borodin and A.O. Ivanova, *Light neighborhoods of* 5*-vertices in* 3*-polytopes with minimum degree* 5, Sib. Èlektron. Mat. Izv. **13** (2016) 584–591. doi:10.17377/semi.2016.13.045

[4] O.V. Borodin, A.O. Ivanova and T.R. Jensen, 5*-stars of low weight in normal plane maps with minimum degree* 5, Discuss. Math. Graph Theory **34** (2014) 539–546. doi:10.7151/dmgt.1748

[5] O.V. Borodin, A.O. Ivanova, O.N. Kazak and E.I. Vasil'eva, *Heights of minor* 5*-stars in* 3*-polytopes with minimum degree* 5 *and no vertices of degree* 6 *and* 7, Discrete Math. **341** (2018) 825–829. doi:10.1016/j.disc.2017.11.021

[6] O.V. Borodin, A.O. Ivanova and D.V. Nikiforov, *Low minor* 5*-stars in* 3*-polytopes with minimum degree* 5 *and no* 6*-vertices*, Discrete Math. **340** (2017) 1612–1616. doi:10.1016/j.disc.2017.03.002

[7] O.V. Borodin, A.O. Ivanova and D.V. Nikiforov, *Low and light minor* 5*-stars in* 3*-polytopes with minimum degree* 5 *and restrictions on the degrees of major vertices*, Sib. Math. J. **58** (2017) 600–605. doi:10.1134/S003744661704005X

[8] O.V. Borodin and D.R. Woodall, *Short cycles of low weight in normal plane maps with minimum degree* 5, Discuss. Math. Graph Theory **18** (1998) 159–164. doi:10.7151/dmgt.1071

[9] P. Franklin, *The four colour problem*, Amer. J. Math. **44** (1922) 225–236. doi:10.2307/2370527

[10] S. Jendrol' and T. Madaras, *On light subgraphs in plane graphs of minimum degree five*, Discuss. Math. Graph Theory **16** (1996) 207–217. doi:10.7151/dmgt.1035

[11] S. Jendrol' and H.-J. Voss, *Light subgraphs of graphs embedded in the plane—A survey*, Discrete Math. **313** (2013) 406–421. doi:10.1016/j.disc.2012.11.007

[12] H. Lebesgue, *Quelques conséquences simples de la formule d'Euler*, J. Math. Pures Appl. **19** (1940) 27–43.

[13] P. Wernicke, *Über den kartographischen Vierfarbensatz*, Math. Ann. **58** (1904) 413–426. doi:10.1007/BF01444968

Received 30 November 2017
Revised 11 June 2018
Accepted 12 June 2018