

LIGHT MINOR 5-STARS IN 3-POLYTOPES WITH MINIMUM DEGREE 5 AND NO 6-VERTICES¹

OLEG V. BORODIN, ANNA O. IVANOVA

AND

EKATERINA I. VASIL'EVA

*Institute of Mathematics Siberian Branch
Russian Academy of Sciences
Novosibirsk, 630090, Russia*

e-mail: brdnoleg@math.nsc.ru
shmgnanna@mail.ru
ekaterinavasilyeva93@gmail.com

Abstract

In 1940, Lebesgue gave an approximate description of the neighborhoods of 5-vertices in the class $\mathbf{P}_5$ of 3-polytopes with minimum degree 5.

Given a 3-polytope P , by $w(P)$ denote the minimum of the degree-sum (weight) of the neighborhoods of 5-vertices (minor 5-stars) in P .

In 1996, Jendrol' and Madaras showed that if a polytope P in $\mathbf{P}_5$ is allowed to have a 5-vertex adjacent to four 5-vertices, then $w(P)$ can be arbitrarily large.

For each P in $\mathbf{P}_5$ without vertices of degree 6 and 5-vertices adjacent to four 5-vertices, it follows from Lebesgue's Theorem that $w(P) \leq 68$. Recently, this bound was lowered to $w(P) \leq 55$ by Borodin, Ivanova, and Jensen and then to $w(P) \leq 51$ by Borodin and Ivanova.

In this note, we prove that every such polytope P satisfies $w(P) \leq 44$, which bound is sharp.

Keywords: planar map, planar graph, 3-polytope, structural properties, 5-star, weight, height.

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