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DECOMPOSING THE COMPLETE GRAPH INTO HAMILTONIAN PATHS (CYCLES) AND 3-STARS

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Abstract

Let H be a graph. A decomposition of H is a set of edge-disjoint subgraphs of H whose union is H. A Hamiltonian path (respectively, cycle) of H is a path (respectively, cycle) that contains every vertex of H exactly once. A k-star, denoted by S_k, is a star with k edges. In this paper, we give necessary and sufficient conditions for decomposing the complete graph into α copies of Hamiltonian path (cycle) and β copies of S_3.

Keywords: decomposition, complete graph, Hamiltonian path, Hamiltonian cycle, star.

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