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EQUATING k MAXIMUM DEGREES IN GRAPHS WITHOUT SHORT CYCLES

Maximilian Fürst, Michael Gentner,

Simon Jäger, Dieter Rautenbach

Institute of Optimization and Operations Research
Ulm University, Germany

e-mail: maximilian.fuerst@uni-ulm.de
michael.gentner@uni-ulm.de
simon.jaeger@uni-ulm.de
dieter.rautenbach@uni-ulm.de

AND

Michael A. Henning

Department of Pure and Applied Mathematics
University of Johannesburg, South Africa

e-mail: mahenning@uj.ac.za

Abstract

For an integer k at least 2, and a graph G, let $f_k(G)$ be the minimum cardinality of a set X of vertices of G such that $G - X$ has either k vertices of maximum degree or order less than k. Caro and Yuster [Discrete Mathematics 310 (2010) 742–747] conjectured that, for every k, there is a constant c_k such that $f_k(G) \leq c_k\sqrt{n(G)}$ for every graph G. Verifying a conjecture of Caro, Lauri, and Zarb [arXiv:1704.08472v1], we show the best possible result that, if t is a positive integer, and F is a forest of order at most $\frac{1}{6}\left(t^3 + 6t^2 + 17t + 12\right)$, then $f_2(F) \leq t$. We study $f_3(F)$ for forests F in more detail obtaining similar almost tight results, and we establish upper bounds on $f_k(G)$ for graphs G of girth at least 5. For graphs G of girth more than $2p$, for p at least 3, our results imply $f_k(G) = O\left(n(G)^{\frac{p+1}{3p}}\right)$. Finally, we show that, for every fixed k, and every given forest F, the value of $f_k(F)$ can be determined in polynomial time.

Keywords: maximum degree, repeated degrees, repetition number.
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