Discussiones Mathematicae
Graph Theory 40 (2020) 841–853
doi:10.7151/dmgt.2152

EQUATING k MAXIMUM DEGREES IN GRAPHS WITHOUT SHORT CYCLES

Maximilian Fürst, Michael Gentner,

Simon Jäger, Dieter Rautenbach

Institute of Optimization and Operations Research
Ulm University, Germany

e-mail: maximilian.fuerst@uni-ulm.de
michael.gentner@uni-ulm.de
simon.jaeger@uni-ulm.de
dieter.rautenbach@uni-ulm.de

and

Michael A. Henning

Department of Pure and Applied Mathematics
University of Johannesburg, South Africa

e-mail: mahenning@uj.ac.za

Abstract

For an integer k at least 2, and a graph G, let $f_k(G)$ be the minimum cardinality of a set X of vertices of G such that $G - X$ has either k vertices of maximum degree or order less than k. Caro and Yuster [Discrete Mathematics 310 (2010) 742–747] conjectured that, for every k, there is a constant c_k such that $f_k(G) \leq c_k\sqrt{n(G)}$ for every graph G. Verifying a conjecture of Caro, Lauri, and Zarb [arXiv:1704.08472v1], we show the best possible result that, if t is a positive integer, and F is a forest of order at most $\frac{1}{6}\left(t^3 + 6t^2 + 17t + 12\right)$, then $f_2(F) \leq t$. We study $f_3(F)$ for forests F in more detail obtaining similar almost tight results, and we establish upper bounds on $f_k(G)$ for graphs G of girth at least 5. For graphs G of girth more than $2p$, for p at least 3, our results imply $f_k(G) = O\left(n(G)^{\frac{p+1}{3p}}\right)$. Finally, we show that, for every fixed k, and every given forest F, the value of $f_k(F)$ can be determined in polynomial time.

Keywords: maximum degree, repeated degrees, repetition number.

2010 Mathematics Subject Classification: 05C05, 05C07.

References

[1] M.O. Albertson and D.L. Boutin, *Lower bounds for constant degree independent sets*, Discrete Math. **127** (1994) 15–21.
doi:10.1016/0012-365X(92)00463-2

[2] B. Bollobás and A.D. Scott, *Independent sets and repeated degrees*, Discrete Math. **170** (1997) 41–49.
doi:10.1016/0012-365X(95)00355-Z

[3] Y. Caro, J. Lauri and C. Zarb, *Equating two maximum degrees*, manuscript.
arXiv:1704.08472v1

[4] Y. Caro, A. Shapira and R. Yuster, *Forcing k-repetitions in degree sequences*, Electron. J. Combin. **21** (2014) #P24.

[5] Y. Caro and R. Yuster, *Large induced subgraphs with equated maximum degree*, Discrete Math. **310** (2010) 742–747.
doi:10.1016/j.disc.2009.09.003

[6] Y. Caro and D.B. West, *Repetition number of graphs*, Electron. J. Combin. **16** (2009) #R7.

[7] M. Miller and J. Širáň, *Moore graphs and beyond: A survey of the degree/diameter problem*, Electron. J. Combin. **20** (2013) #DS14v2.

Received 29 November 2017
Revised 15 May 2018
Accepted 15 May 2018