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A NOTE ON LOWER BOUNDS FOR INDUCED RAMSEY NUMBERS

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Abstract

We say that a graph F *strongly arrows* a pair of graphs (G,H) and write $F\overset{\text{ind}}{\longrightarrow}(G,H)$ if any 2-coloring of its edges with red and blue leads to either a red G or a blue H appearing as induced subgraphs of F. *The induced Ramsey number*, $IR(G,H)$ is defined as $\min\{|V(F)| : F\overset{\text{ind}}{\longrightarrow}(G,H)\}$. We will consider two aspects of induced Ramsey numbers. Firstly we will show that the lower bound of the induced Ramsey number for a connected graph G with independence number α and a graph H with clique number ω is roughly $\frac{\omega^2\alpha}{2}$. This bound is sharp. Moreover we will also consider the case when G is not connected providing also a sharp lower bound which is linear in both parameters.

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References

[1] J. Beck, *On the size Ramsey number of paths, trees and circuits* I, J. Graph Theory **7** (1983) 115–129.
doi:10.1002/jgt.3190070115

[2] D. Conlon, J. Fox and B. Sudakov, *On two problems in graph Ramsey theory*, Combinatorica **32** (2012) 513–535.
doi:10.1007/s00493-012-2710-3

[3] V. Chvátal and F. Harary, *Generalized Ramsey theory for graphs.* III. *Small off-diagonal numbers*, Pacific J. Math. **41** (1972) 335–345.
doi:10.2140/pjm.1972.41.335

[4] V. Chvátal, *Tree-complete graph Ramsey numbers*, J. Graph Theory **1** (1977) 93.
doi:10.1002/jgt.3190010118

[5] W. Deuber, *A generalization of Ramsey's theorem*, in: Infinite and Finite Sets, R. Rado A. Hajnal and V. Sós, (Eds.) **10** (North-Holland, Amsterdam, 1975) 323–332.

[6] P. Erdős, *Some remarks on the theory of graphs*, Bull. Amer. Math. Soc. **53** (1947) 292–294
doi:10.1090/S0002-9904-1947-08785-1

[7] P. Erdős, *On some problems in graph theory, combinatorial analysis and combinatorial number theory*, in: Proc. Conf. Hon. P. Erdös, Cambridge 1983, Graph Theory and Combinatorics (Academic Press, New York, 1984) 1–17.

[8] P. Erdős, A. Hajnal and L. Pósa, *Strong embeddings of graphs into colored graphs*, in: Infinite and Finite Sets, R. Rado A. Hajnal and V. Sós, (Eds.) **10** (North-Holland, 1975) 585–595.

[9] I. Gorgol, *A note on a triangle-free-complete graph induced Ramsey number*, Discrete Math. **235** (2001) 159–163.
doi:10.1016/S0012-365X(00)00269-7

[10] I. Gorgol and T. Łuczak, *On induced Ramsey numbers*, Discrete Math. **251** (2002) 87–96.
doi:10.1016/S0012-365X(01)00328-4

[11] F. Harary, J. Nešetřil and V. Rödl, *Generalized Ramsey theory for graphs.* XIV. *Induced Ramsey numbers*, in: Proceedings of the Third Czechoslovak Symposium on Graph Theory, Praque 1982, Graphs and Other Combinatorial Topics **59** (1983) 90–100.

[12] P. Haxell, Y. Kohayakawa and T. Łuczak, *The induced size-Ramsey number of cycles*, Combin. Probab. Comput. **4** (1995) 217–239.
doi:10.1017/S0963548300001619

[13] Y. Kohayakawa, H.J. Prömel and V. Rödl, *Induced Ramsey numbers*, Combinatorica **18** (1998) 373–404.
doi:10.10071PL00009828

[14] A. Kostochka and N. Sheikh, *On the induced Ramsey number $IR(P3, H)$*, Topics in Discrete Mathematics **26** (2006) 155–167.
doi:10.1007/3-540-33700-8_10

[15] T. Łuczak and V. Rödl, *On induced Ramsey numbers for graphs with bounded maximum degree*, J. Combin. Theory Ser. B **66** (1996) 324–333.
doi:10.1006/jctb.1996.0025

[16] V. Rödl, The Dimention of a Graph and Generalized Ramsey Theorems, Master's Thesis (Charles University, Prague, 1973).

[17] V. Rödl, *A generalization of Ramsey theorem*, in: Proc. Symp. Comb. Anal., Zielona Góra 1976, Graphs, Hypergraphs and Block Systems (1976) 211–219.

[18] M. Schaefer and P. Shah, *Induced graph Ramsey theory*, Ars Combin. **66** (2003) 3–21.

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