

2-SPANNING CYCLABILITY PROBLEMS OF SOME GENERALIZED PETERSEN GRAPHS

MENG-CHIEN YANG, LIH-HSING HSU

*Department of Computer Science and Communication Engineering
Providence University, Taichung, Taiwan 43301, R.O.C.*

e-mail: mcyang2@pu.edu.tw
lihhsing@gmail.com

CHUN-NAN HUNG

*Department of Information Management, Da-Yeh University
No.168, University Rd., Dacun, Changhua, Taiwan 51591, R.O.C.*

e-mail: spring@mail.dyu.edu.tw

AND

EDDIE CHENG

*Department of Mathematics and Statistics
Oakland University, Rochester, MI 48309*

e-mail: echeng@oakland.edu

Abstract

A graph G is called r -spanning cyclable if for every r distinct vertices $v_1, v_2, \dots, v_r$ of G , there exists r cycles $C_1, C_2, \dots, C_r$ in G such that v_i is on C_i for every i , and every vertex of G is on exactly one cycle C_i . In this paper, we consider the 2-spanning cyclable problem for the generalized Petersen graph $GP(n, k)$. We solved the problem for $k \leq 4$. In addition, we provide an additional observation for general k as well as stating a conjecture.

Keywords: Petersen graph, spanning cyclable.

2010 Mathematics Subject Classification: 05C38, 05C45.

REFERENCES

- [1] M. Albert, R.E.L. Aldred and D. Holton, *On 3*-connected graphs*, Australas. J. Combin. **24** (2001) 193–207.

- [2] B. Alspach, *The classification of Hamiltonian generalized Petersen graphs*, J. Combin. Theory Ser. B **34** (1983) 293–312.
doi:10.1016/0095-8956(83)90042-4
- [3] B. Alspach, D. Bryant and D. Dyer, *Paley graphs have Hamilton decompositions*, Discrete Math. **312** (2012) 113–118.
doi:10.1016/j.disc.2011.06.003
- [4] B. Alspach and J. Liu, *On the Hamilton connectivity of generalized Petersen graphs*, Discrete Math. **309** (2009) 5461–5473.
doi:10.1016/j.disc.2008.12.016
- [5] M. Behzad, P. Hatami and E.S. Mahmoodian, *Minimum vertex covers in the generalized Petersen graphs $P(n, 2)$* , Bull. Inst. Combin. Appl. **56** (2009) 98–102.
- [6] J.A. Bondy, *Pancyclic graphs I*, J. Combin. Theory Ser. B **11** (1971) 80–84.
doi:10.1016/0095-8956(71)90016-5
- [7] J.A. Bondy, *Variations on the Hamiltonian theme*, Canad. Math. Bull. **15** (1972) 57–62.
doi:10.4153/CMB-1972-012-3
- [8] M.Y. Chan and S.J. Lee, *On the existence of Hamiltonian circuits in faulty hypercubes*, SIAM J. Discrete Math. **4** (1991) 511–527.
doi:10.1137/0404045
- [9] R.J. Faundree, *Survey of results on k -ordered graphs*, Discrete Math. **229** (2001) 73–87.
doi:10.1016/S0012-365X(00)00202-8
- [10] J.R. Faundree, R.J. Gould, A.V. Kostochka, L. Lesniak, I. Schiermeyer and A. Saito, *Degree conditions for k -ordered Hamiltonian graphs*, J. Graph Theory **42** (2003) 199–210.
doi:10.1002/jgt.10084
- [11] S. Fujita and T. Araki, *Three-round adaptive diagnosis in binary n -cubes*, Lecture Notes in Comput. Sci. **3341** (2004) 442–451.
doi:10.1007/978-3-540-30551-4_39
- [12] S.L. Hakimi and E.F. Schmeichel, *On the number of cycles of length k in a maximal planar graph*, J. Graph Theory **3** (1979) 69–86.
doi:10.1002/jgt.3190030108
- [13] C.-N. Hung, D. Lu, R. Jia, C.-K. Lin, L. Lipták, E. Cheng, J.J.M. Tan and L.-H. Hsu, *4-ordered Hamiltonian problems for the generalized Petersen graph $GP(n, 4)$* , Math. Comput. Modelling **57** (2013) 595–601.
doi:10.1016/j.mcm.2012.07.022
- [14] S.Y. Hsieh, G.H. Chen and C.W. Ho, *Fault-free Hamiltonian cycles in faulty arrangement graphs*, IEEE Trans. Parallel Distributed Systems **10** (1999) 223–237.
doi:10.1109/71.755822
- [15] L.-H. Hsu and C.-K. Lin, *Graph Theory and Interconnection Networks* (CRC Press, 2009).

- [16] L.-H. Hsu, J.M. Tan, E. Cheng, L. Lipták, C.K. Lin and M. Tsai, *Solution to an open problem of 4-ordered Hamiltonian graphs*, Discrete Math. **312** (2012) 2356–2370.
doi:10.1016/j.disc.2012.04.003
- [17] M. Lewinter and W. Widulski, *Hyper-Hamilton laceable and caterpillar-spannable product graphs*, Comput. Math. Appl. **34** (1997) 99–104.
doi:10.1016/S0898-1221(97)00223-X
- [18] R. Li, S. Li and Y. Guo, *Degree conditions on distance 2 vertices that imply k -ordered Hamiltonian*, Discrete Appl. Math. **158** (2010) 331–339.
doi:10.1016/j.dam.2009.05.005
- [19] C.-K. Lin, H.-M. Huang and L.-H. Hsu, *The super connectivity of the pancake graphs and super laceability of the star graphs*, Theoret. Comput. Sci. **339** (2005) 257–271.
doi:10.1016/j.tcs.2005.02.007
- [20] C.-K. Lin, H.-M. Huang, J.J.M. Tan and L.-H. Hsu, *On spanning connected graphs*, Discrete Math. **308** (2008) 1330–1333.
doi:10.1016/j.disc.2007.03.072
- [21] J. Liu, *Hamiltonian decompositions of Cayley graphs on Abelian groups*, Discrete Math. **131** (1994) 163–171.
doi:10.1016/0012-365X(94)90381-6
- [22] J. Liu, *Hamiltonian decompositions of Cayley graphs on abelian groups of even order*, J. Combin. Theory Ser. B **88** (2003) 305–321.
doi:10.1016/S0095-8956(03)00033-9
- [23] K. Mészáros, *On 3-regular 4-ordered graphs*, Discrete Math. **308** (2008) 2149–2155.
doi:10.1016/j.disc.2007.04.061
- [24] L. Ng and M. Schultz, *k -ordered Hamiltonian graphs*, J. Graph Theory **24** (1997) 45–57.
doi:10.1002/(SICI)1097-0118(199701)24:1<45::AID-JGT6>3.0.CO;2-J
- [25] G.N. Robertson, *Graphs Minimal under Girth, Valency and Connectivity Constraints*, PhD Thesis (University of Waterloo, 1968).
- [26] D.R. Silaban, A. Parestu, B.N. Herawati, K.A. Sugeng and Slamin, *Vertex-magic total labelings of union of generalized Petersen graphs and union of special circulant graphs*, J. Combin. Math. Combin. Comput. **71** (2009) 201–207.
- [27] C. Tong, X. Lin, Y. Yang and M. Luo, *2-rainbow domination of generalized Petersen graphs $P(n, 2)$* , Discrete Appl. Math. **157** (2009) 1932–1937.
doi:10.1016/j.dam.2009.01.020
- [28] M.E. Watkins, *A theorem on Tait colorings with an application to the generalized Petersen graphs*, J. Combin. Theory **6** (1969) 152–164.
doi:10.1016/S0021-9800(69)80116-X
- [29] G. Xu, *2-rainbow domination in generalized Petersen graphs $P(n, 3)$* , Discrete Appl. Math. **157** (2009) 2570–2573.
doi:10.1016/j.dam.2009.03.016

Received 18 September 2017

Revised 5 March 2018

Accepted 5 March 2018