

## REGULAR COLORINGS IN REGULAR GRAPHS

ANTON BERNSHTEYN

*Department of Mathematics, University of Illinois at Urbana-Champaign*

**e-mail:** bernsht2@illinois.edu

OMID KHORMALI

*Department of Mathematical Sciences, University of Montana*

**e-mail:** omid.khormali@umconnect.umt.edu

RYAN R. MARTIN

*Department of Mathematics, Iowa State University*

**e-mail:** rymartin@iastate.edu

JONATHAN ROLLIN

*Department of Mathematics, Karlsruhe Institute of Technology*

**e-mail:** jonathan.rollin@kit.edu

DANNY RORABAUGH

*Department of Mathematics and Statistics, Queen's University*

**e-mail:** rorabaug@email.sc.edu

SONGLING SHAN

*Department of Mathematics, Vanderbilt University*

**e-mail:** songling.shan@vanderbilt.edu

AND

ANDREW J. UZZELL

*Mathematics and Statistics Department, Grinnell College*

**e-mail:** uzzellan@grinnell.edu

### Abstract

An  $(r - 1, 1)$ -coloring of an  $r$ -regular graph  $G$  is an edge coloring (with arbitrarily many colors) such that each vertex is incident to  $r - 1$  edges of one color and 1 edge of a different color. In this paper, we completely characterize all 4-regular pseudographs (graphs that may contain parallel edges and loops) which do not have a  $(3, 1)$ -coloring. Also, for each  $r \geq 6$  we construct graphs that are not  $(r - 1, 1)$ -colorable and, more generally, are not  $(r - t, t)$ -colorable for small  $t$ .

**Keywords:** edge coloring, graph factors, regular graphs.

**2010 Mathematics Subject Classification:** 05C15.

### REFERENCES

- [1] S. Akbari and M. Kano,  $\{k, r - k\}$ -factors of  $r$ -regular graphs, *Graphs Combin.* **30** (2014) 821–826.  
doi:10.1007/s00373-013-1324-x
- [2] J. Akiyama and M. Kano, *Factors and Factorizations of Graphs: Proof Techniques in Factor Theory* (Springer, Heidelberg, 2011).  
doi:10.1007/978-3-642-21919-1
- [3] M. Axenovich and J. Rollin, *Brooks type results for conflict-free colorings and  $\{a, b\}$ -factors in graphs*, *Discrete Math.* **338** (2015) 2295–2301.  
doi:10.1016/j.disc.2015.05.020
- [4] A.Y. Bernshteyn, *3-regular subgraphs and  $(3, 1)$ -colorings of 4-regular pseudographs*, *J. Appl. Ind. Math.* **8** (2014) 458–466.  
doi:10.1134/S1990478914040024
- [5] B. Bollobás, A. Saito and N.C. Wormald, *Regular factors of regular graphs*, *J. Graph Theory* **9** (1985) 97–103.  
doi:10.1002/jgt.3190090107
- [6] J.A. Bondy and U.S.R. Murty, *Graph Theory with Applications* (American Elsevier Publishing Co., New York, 1976).
- [7] F.R.K. Chung and R.L. Graham, *Recent results in graph decompositions*, in: *Combinatorics Proc. 8th British Combinatorial Conference* (Swansea, 1981), London Math. Soc. Lecture Note Ser. **52** (1981) 103–123.
- [8] L. Lovász, *The factorization of graphs. II*, *Acta Math. Hungar.* **23** (1972) 223–246.  
doi:10.1007/BF01889919
- [9] H. Lu, D.G.L. Wang and Q. Yu, *On the existence of general factors in regular graphs*, *SIAM J. Discrete Math.* **27** (2013) 1862–1869.  
doi:10.1137/120895792
- [10] J. Petersen, *Die Theorie der regulären graphs*, *Acta Math.* **15** (1891) 191–220.  
doi:10.1007/BF02392606

- [11] M.D. Plummer, *Graph factors and factorization: 1985–2003: A survey*, Discrete Math. **307** (2007) 791–821.  
doi:10.1016/j.disc.2005.11.059
- [12] V.A. Tashkinov, *3-regular parts of 4-regular graphs*, Mat. Zametki **36** (1984) 239–259.
- [13] V.A. Tashkinov, *Regular parts of regular pseudographs*, Mat. Zametki **43** (1988) 263–275.
- [14] W.T. Tutte, *The subgraph problem*, Ann. Discrete Math. **3** (1978) 289–295.  
doi:10.1016/S0167-5060(08)70514-4

Received 4 October 2017

Revised 7 May 2018

Accepted 10 May 2018