Discussiones Mathematicae
Graph Theory 40 (2020) 795–806
doi:10.7151/dmgt.2149

REGULAR COLORINGS IN REGULAR GRAPHS

Anton Bernshteyn

Department of Mathematics, University of Illinois at Urbana-Champaign
e-mail: bernsht2@illinois.edu

Omid Khormali

Department of Mathematical Sciences, University of Montana
e-mail: omid.khormali@umconnect.umt.edu

Ryan R. Martin

Department of Mathematics, Iowa State University
e-mail: rymartin@iastate.edu

Jonathan Rollin

Department of Mathematics, Karlsruhe Institute of Technology
e-mail: jonathan.rollin@kit.edu

Danny Rorabaugh

Department of Mathematics and Statistics, Queen's University
e-mail: rorabaug@email.sc.edu

Songling Shan

Department of Mathematics, Vanderbilt University
e-mail: songling.shan@vanderbilt.edu

and

Andrew J. Uzzell

Mathematics and Statistics Department, Grinnell College
e-mail: uzzellan@grinnell.edu

Abstract

An $(r-1,1)$-*coloring* of an r-regular graph G is an edge coloring (with arbitrarily many colors) such that each vertex is incident to $r-1$ edges of one color and 1 edge of a different color. In this paper, we completely characterize all 4-regular pseudographs (graphs that may contain parallel edges and loops) which do not have a $(3,1)$-coloring. Also, for each $r \geq 6$ we construct graphs that are not $(r-1,1)$-colorable and, more generally, are not $(r-t,t)$-colorable for small t.

Keywords: edge coloring, graph factors, regular graphs.

2010 Mathematics Subject Classification: 05C15.

References

[1] S. Akbari and M. Kano, *$\{k, r-k\}$-factors of r-regular graphs*, Graphs Combin. **30** (2014) 821–826. doi:10.1007/s00373-013-1324-x

[2] J. Akiyama and M. Kano, Factors and Factorizations of Graphs: Proof Techniques in Factor Theory (Springer, Heidelberg, 2011). doi:10.1007/978-3-642-21919-1

[3] M. Axenovich and J. Rollin, *Brooks type results for conflict-free colorings and $\{a,b\}$-factors in graphs*, Discrete Math. **338** (2015) 2295–2301. doi:10.1016/j.disc.2015.05.020

[4] A.Y. Bernshteyn, *3-regular subgraphs and $(3,1)$-colorings of 4-regular pseudographs*, J. Appl. Ind. Math. **8** (2014) 458–466. doi:10.1134/S1990478914040024

[5] B. Bollobás, A. Saito and N.C. Wormald, *Regular factors of regular graphs*, J. Graph Theory **9** (1985) 97–103. doi:10.1002/jgt.3190090107

[6] J.A. Bondy and U.S.R. Murty, Graph Theory with Applications (American Elsevier Publishing Co., New York, 1976).

[7] F.R.K. Chung and R.L. Graham, *Recent results in graph decompositions*, in: Combinatorics Proc. 8th British Combinatorial Conference (Swansea, 1981), London Math. Soc. Lecture Note Ser. **52** (1981) 103–123.

[8] L. Lovász, *The factorization of graphs.* II, Acta Math. Hungar. **23** (1972) 223–246. doi:10.1007/BF01889919

[9] H. Lu, D.G.L. Wang and Q. Yu, *On the existence of general factors in regular graphs*, SIAM J. Discrete Math. **27** (2013) 1862–1869. doi:10.1137/120895792

[10] J. Petersen, *Die Theorie der regulären graphs*, Acta Math. **15** (1891) 191–220. doi:10.1007/BF02392606

[11] M.D. Plummer, *Graph factors and factorization*: 1985–2003: *A survey*, Discrete Math. **307** (2007) 791–821.
doi:10.1016/j.disc.2005.11.059

[12] V.A. Tashkinov, 3-*regular parts of* 4-*regular graphs*, Mat. Zametki **36** (1984) 239–259.

[13] V.A. Tashkinov, *Regular parts of regular pseudographs*, Mat. Zametki **43** (1988) 263–275.

[14] W.T. Tutte, *The subgraph problem*, Ann. Discrete Math. **3** (1978) 289–295.
doi:10.1016/S0167-5060(08)70514-4

Received 4 October 2017
Revised 7 May 2018
Accepted 10 May 2018