

HAMILTONICITIES OF DOUBLE DOMINATION CRITICAL AND STABLE CLAW-FREE GRAPHS

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Abstract

A graph G with the double domination number $\gamma_{\times 2}(G) = k$ is said to be k - $\gamma_{\times 2}$ -critical if $\gamma_{\times 2}(G + uv) < k$ for any $uv \notin E(G)$. On the other hand, a graph G with $\gamma_{\times 2}(G) = k$ is said to be k - $\gamma_{\times 2}^+$ -stable if $\gamma_{\times 2}(G + uv) = k$ for any $uv \notin E(G)$ and is said to be k - $\gamma_{\times 2}^-$ -stable if $\gamma_{\times 2}(G - uv) = k$ for any $uv \in E(G)$. The problem of interest is to determine whether or not 2-connected k - $\gamma_{\times 2}$ -critical graphs are Hamiltonian. In this paper, for $k \geq 4$, we provide a 2-connected k - $\gamma_{\times 2}$ -critical graph which is non-Hamiltonian. We prove that all 2-connected k - $\gamma_{\times 2}$ -critical claw-free graphs are Hamiltonian when $2 \leq k \leq 5$. We show that the condition claw-free when $k = 4$ is best possible. We further show that every 3-connected k - $\gamma_{\times 2}$ -critical claw-free graph is Hamiltonian when $2 \leq k \leq 7$. We also investigate Hamiltonian properties of k - $\gamma_{\times 2}^+$ -stable graphs and k - $\gamma_{\times 2}^-$ -stable graphs.

Keywords: double domination, critical, stable, Hamiltonian.

2010 Mathematics Subject Classification: 05C69, 05C45.

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Received 30 October 2017

Revised 7 May 2018

Accepted 7 May 2018