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HAMILTONICITIES OF DOUBLE DOMINATION CRITICAL AND STABLE CLAW-FREE GRAPHS

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Abstract

A graph G with the double domination number $\gamma_{\times 2}(G) = k$ is said to be k-$\gamma_{\times 2}$-critical if $\gamma_{\times 2}(G + uv) < k$ for any $uv \notin E(G)$. On the other hand, a graph G with $\gamma_{\times 2}(G) = k$ is said to be k-$\gamma^{+}_{\times 2}$-stable if $\gamma_{\times 2}(G + uv) = k$ for any $uv \notin E(G)$ and is said to be k-$\gamma^{-}_{\times 2}$-stable if $\gamma_{\times 2}(G - uv) = k$ for any $uv \in E(G)$. The problem of interest is to determine whether or not 2-connected k-$\gamma_{\times 2}$-critical graphs are Hamiltonian. In this paper, for $k \geq 4$, we provide a 2-connected k-$\gamma_{\times 2}$-critical graph which is non-Hamiltonian. We prove that all 2-connected k-$\gamma_{\times 2}$-critical claw-free graphs are Hamiltonian when $2 \leq k \leq 5$. We show that the condition claw-free when $k = 4$ is best possible. We further show that every 3-connected k-$\gamma_{\times 2}$-critical claw-free graph is Hamiltonian when $2 \leq k \leq 7$. We also investigate Hamiltonian properties of k-$\gamma^{+}_{\times 2}$-stable graphs and k-$\gamma^{-}_{\times 2}$-stable graphs.

Keywords: double domination, critical, stable, Hamiltonian.

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