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ON FACTORABLE BIGRAPHIC PAIRS [1]

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Abstract

Let $S = (a_1, \ldots, a_m;\ b_1, \ldots, b_n)$, where $a_1, \ldots, a_m$ and $b_1, \ldots, b_n$ are two sequences of nonnegative integers. We say that S is a *bigraphic pair* if there exists a simple bipartite graph G with partite sets $\{x_1, x_2, \ldots, x_m\}$ and $\{y_1, y_2, \ldots, y_n\}$ such that $d_G(x_i) = a_i$ for $1 \le i \le m$ and $d_G(y_j) = b_j$ for $1 \le j \le n$. In this case, we say that G is a *realization* of S. Analogous to Kundu's k-factor theorem, we show that if $(a_1, a_2, \ldots, a_m; b_1, b_2, \ldots, b_n)$ and $(a_1 - e_1, a_2 - e_2, \ldots, a_m - e_m;\ b_1 - f_1, b_2 - f_2, \ldots, b_n - f_n)$ are two bigraphic pairs satisfying $k \le f_i \le k + 1$, $1 \le i \le n$ (or $k \le e_i \le k + 1$, $1 \le i \le m$), for some $0 \le k \le m - 1$ (or $0 \le k \le n - 1$), then $(a_1, a_2, \ldots, a_m;$ $b_1, b_2, \ldots, b_n)$ has a realization containing an $(e_1, e_2, \ldots, e_m;\ f_1, f_2, \ldots, f_n)$-factor. For $m = n$, we also give a necessary and sufficient condition for an $(k^n; k^n)$-factorable bigraphic pair to be connected $(k^n; k^n)$-factorable when $k \ge 2$. This implies a characterization of bigraphic pairs with a realization containing a Hamiltonian cycle.

Keywords: degree sequence, bigraphic pair, Hamiltonian cycle.

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