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ON SOME PROPERTIES OF ANTIPODAL PARTIAL CUBES

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Abstract

We prove that an antipodal bipartite graph is a partial cube if and only it is interval monotone. Several characterizations of the principal cycles of an antipodal partial cube are given. We also prove that an antipodal partial cube G is a prism over an even cycle if and only if its order is equal to $4(\mathrm{diam}(G) - 1)$, and that the girth of an antipodal partial cube is less than its diameter whenever it is not a cycle and its diameter is at least equal to 6.

Keywords: antipodal graph, partial cube, interval monotony, girth, diameter.

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References

[1] A. Berman, A. Kotzig and G. Sabidussi, *Antipodal graphs of diameter* 4 *and extremal girth*, in: Contemporary Methods in Graph Theory, R. Bodendiek (Ed(s)), (B.I. Wissenschaftsverlag, Mannheim, 1990) 137–150.

[2] B. Brešar and S. Klavžar, *On partial cubes and graphs with convex intervals*, Comment. Math. Univ. Carolin. **43** (2002) 537–545.

[3] V. Chepoi, *Isometric subgraphs of Hamming graphs and d-convexity*, Cybernetics **24** (1988) 6–11.
doi:10.1007/BF01069520

[4] V. Chepoi, K. Knauer and T. Marc, *Partial cubes without Q_3^- minors*, CoRR abs/1606.02154 (2016) (submitted).

[5] J. Desharnais, *Maille et plongements de graphes antipodaux*, Mémoire de maitrise (Université de Montréal, 1993).

[6] D. Djoković, *Distance-preserving subgraphs of hypercubes*, J. Combin. Theory Ser. B **14** (1973) 263–267.
doi:10.1016/0095-8956(73)90010-5

[7] V.V. Firsov, *Isometric embedding of a graph in a Boolean cube*, Cybernet. Systems Anal. **1** (1965) 112–113.
doi:10.1007/BF01074705

[8] K. Fukuda and K. Handa, *Antipodal graphs and oriented matroids*, Discrete Math. **111** (1993) 245–256.
doi:10.1016/0012-365X(93)90159-Q

[9] F. Glivjak, A. Kotzig and J. Plesník, *Remarks on the graphs with a central symmetry*, Monatsh.Math. **74** (1970) 302–307.
doi:10.1007/BF01302697

[10] F. Göbel and H.J. Veldman, *Even graphs*, J. Graph Theory **10** (1986) 225–239.
doi:10.1002/jgt.3190100212

[11] R. Hammack, W. Imrich and S. Klavžar, Handbook of Product Graphs, Second Edition (CRC Press, 2011).

[12] S. Klavžar and M. Kovše, *On even and harmonic-even partial cubes*, Ars Combin. **93** (2009) 77–86.

[13] K. Knauer and T. Marc, *On tope graphs of complexes of oriented matroids*, arXiv preprint, (2017).
arXiv:1701.05525

[14] A. Kotzig, *Centrally symmetric graphs*, Czechoslovak Math. J. **18** (1968) 606–615.

[15] A. Kotzig and P. Laufer, Generalized S-graphs, Research Report CRM-779 (Centre de recherches mathématiques, Université de Montréal, 1978).

[16] T. Marc, *There are no finite partial cubes of girth more than* 6 *and minimum degree at least* 3, European J. Combin. **55** (2016) 62–72.
doi:10.1016/j.ejc.2016.01.005

[17] H.M. Mulder, *The structure of median graphs*, Discrete Math. **24** (1978) 197–204.
doi:10.1016/0012-365X(78)90199-1

[18] S. Ovchinnikov, *Partial cubes: structures, characterizations, and constructions*, Discrete Math. **308** (2008) 5597–5621.
doi:10.1016/j.disc.2007.10.025

[19] N. Polat, *Netlike partial cubes* I. *General properties*, Discrete Math. **307** (2007) 2704–2722.
doi:10.1016/j.disc.2007.01.018

[20] N. Polat, *On some characterizations of antipodal partial cubes*, Discuss. Math. Graph Theory, to appear.
doi:10.7151/dmgt.2083

[21] G. Sabidussi, *Graphs without dead ends*, European J. Combin. **17** (1996) 69–87.
doi:10.1006/eujc.1996.0006

[22] C. Tardif, *Planar antipodal bipartite graphs*, manuscript (1991).

[23] P. Winkler, *Isometric embedding in products of complete graphs*, Discrete Appl. Math. **7** (1984) 221–225.
doi:10.1016/0166-218X(84)90069-6

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