

ON SOME PROPERTIES OF ANTIPODAL PARTIAL CUBES

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Abstract

We prove that an antipodal bipartite graph is a partial cube if and only if it is interval monotone. Several characterizations of the principal cycles of an antipodal partial cube are given. We also prove that an antipodal partial cube G is a prism over an even cycle if and only if its order is equal to $4(\text{diam}(G) - 1)$, and that the girth of an antipodal partial cube is less than its diameter whenever it is not a cycle and its diameter is at least equal to 6.

Keywords: antipodal graph, partial cube, interval monotony, girth, diameter.

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