Discussiones Mathematicae
Graph Theory 40 (2020) 331–344
doi:10.7151/dmgt.2145

NEIGHBOR SUM DISTINGUISHING TOTAL CHOOSABILITY OF IC-PLANAR GRAPHS

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Abstract

Two distinct crossings are independent if the end-vertices of the crossed pair of edges are mutually different. If a graph G has a drawing in the plane such that every two crossings are independent, then we call G a plane graph with independent crossings or IC-planar graph for short. A proper total-k-coloring of a graph G is a mapping $c : V(G) \cup E(G) \to \{1, 2, \ldots, k\}$ such that any two adjacent elements in $V(G) \cup E(G)$ receive different colors. Let $\sum_c(v)$ denote the sum of the color of a vertex v and the colors of all incident edges of v. A total-k-neighbor sum distinguishing-coloring of G is a total-k-coloring of G such that for each edge $uv \in E(G)$, $\sum_c(u) \neq \sum_c(v)$. The least number k needed for such a coloring of G is the neighbor sum distinguishing total chromatic number, denoted by $\chi''_\Sigma(G)$. In this paper, it is proved that if G is an IC-planar graph with maximum degree $\Delta(G)$, then $ch''_\Sigma(G) \leq \max\{\Delta(G) + 3, 17\}$, where $ch''_\Sigma(G)$ is the neighbor sum distinguishing total choosability of G.

Keywords: neighbor sum distinguishing total choosability, maximum degree, IC-planar graph, Combinatorial Nullstellensatz.

2010 Mathematics Subject Classification: 05C15.

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Received 30 June 2017
Revised 21 March 2018
Accepted 21 March 2018