

NEIGHBOR SUM DISTINGUISHING TOTAL CHOOSABILITY OF IC-PLANAR GRAPHS

WEN-YAO SONG, LIAN-YING MIAO

School of Mathematics
China University of Mining and Technology
Xuzhou 221116, P.R. China

e-mail: songwenyao@cumt.edu.cn
miaolianyong@cumt.edu.cn

AND

YUAN-YUAN DUAN

School of Mathematics and Statistics
Zaozhuang University
Zaozhuang 277160, P.R. China

e-mail: duanyy0827@sina.com

Abstract

Two distinct crossings are independent if the end-vertices of the crossed pair of edges are mutually different. If a graph G has a drawing in the plane such that every two crossings are independent, then we call G a plane graph with independent crossings or IC-planar graph for short. A proper total- k -coloring of a graph G is a mapping $c : V(G) \cup E(G) \rightarrow \{1, 2, \dots, k\}$ such that any two adjacent elements in $V(G) \cup E(G)$ receive different colors. Let $\sum_c(v)$ denote the sum of the color of a vertex v and the colors of all incident edges of v . A total- k -neighbor sum distinguishing-coloring of G is a total- k -coloring of G such that for each edge $uv \in E(G)$, $\sum_c(u) \neq \sum_c(v)$. The least number k needed for such a coloring of G is the neighbor sum distinguishing total chromatic number, denoted by $\chi''_{\Sigma}(G)$. In this paper, it is proved that if G is an IC-planar graph with maximum degree $\Delta(G)$, then $ch''_{\Sigma}(G) \leq \max\{\Delta(G) + 3, 17\}$, where $ch''_{\Sigma}(G)$ is the neighbor sum distinguishing total choosability of G .

Keywords: neighbor sum distinguishing total choosability, maximum degree, IC-planar graph, Combinatorial Nullstellensatz.

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