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ROMAN {2}-BONDAGE NUMBER OF A GRAPH

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Abstract

For a given graph $G=(V,E)$, a Roman $\{2\}$-dominating function $f : V(G) \to \{0,1,2\}$ has the property that for every vertex u with $f(u)=0$, either u is adjacent to a vertex assigned 2 under f, or is adjacent to at least two vertices assigned 1 under f. The Roman $\{2\}$-domination number of G, $\gamma_{\{R2\}}(G)$, is the minimum of $\sum_{u\in V(G)} f(u)$ over all such functions. In this paper, we initiate the study of the problem of finding Roman $\{2\}$-bondage number of G. The Roman $\{2\}$-bondage number of G, $b_{\{R2\}}$, is defined as the cardinality of a smallest edge set $E' \subseteq E$ for which $\gamma_{\{R2\}}(G-E') > \gamma_{\{R2\}}(G)$. We first demonstrate complexity status of the problem by proving that the problem is NP-Hard. Then, we derive useful parametric as well as fixed upper bounds on the Roman $\{2\}$-bondage number of G. Specifically, it is known that the Roman bondage number of every planar graph does not exceed 15 (see [S. Akbari, M. Khatirinejad and S. Qajar, *A note on the Roman bondage number of planar graphs*, Graphs Combin. 29 (2013) 327–331]). We show that same bound will be preserved while computing the Roman $\{2\}$-bondage number of such graphs. The paper is then concluded by computing exact value of the parameter for some classes of graphs.

Keywords: domination, Roman $\{2\}$-domination, Roman $\{2\}$-bondage number.

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