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LOWER BOUND ON THE NUMBER OF HAMILTONIAN CYCLES OF GENERALIZED PETERSEN GRAPHS

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Abstract

In this paper, we investigate the number of Hamiltonian cycles of a generalized Petersen graph $P(N,k)$ and prove that

$$\Psi\big(P(N,3)\big) \geqslant N \cdot \alpha_N,$$

where $\Psi\big(P(N,3)\big)$ is the number of Hamiltonian cycles of $P(N,3)$ and α_N satisfies that for any $\varepsilon > 0$, there exists a positive integer M such that when $N > M$,

$$\left((1-\varepsilon)\frac{(1-r^3)}{6r^3+5r^2+3}\right)\left(\frac{1}{r}\right)^{N+2} < \alpha_N < \left((1+\varepsilon)\frac{(1-r^3)}{6r^3+5r^2+3}\right)\left(\frac{1}{r}\right)^{N+2},$$

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where $\frac{1}{r} = \max\left\{\left|\frac{1}{r_j}\right| : j = 1, 2, \ldots, 6\right\}$ and each r_j is a root of equation $x^6 + x^5 + x^3 - 1 = 0$, $r \approx 0.782$. This shows that $\Psi\big(P(N,3)\big)$ is exponential in N and also deduces that the number of 1-factors of $P(N,3)$ is exponential in N.

Keywords: generalized Petersen graph, Hamiltonian cycle, partition number, 1-factor.

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